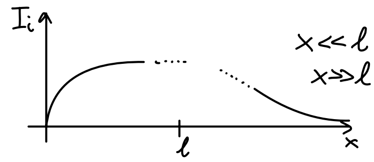
$l = 5\ cm$

$B = 1T$

$x(I_{max}) = ?$

$U_i = R \cdot I_i \ ; \ R = \xi \frac{2(x+l)}{S}$ $\qquad v^2 = v_0^2 + 2ax$

$\rightarrow I_i = \frac{U_i}{R} = \frac{U_i S}{\xi 2(x+l)} = Blv(x)\frac{S}{\xi 2(x+l)} = Bl\sqrt{2ax}\,\frac{S}{\xi 2(x+l)} \propto \frac{\sqrt{x}}{x+l}$



$x \ll l \rightarrow I_i \propto \sqrt{x}$

$x \gg l \rightarrow I_i \propto \frac{1}{\sqrt{x}}$

$\left(\frac{\sqrt{x}}{x+l}\right)' = \frac{\frac{1}{2\sqrt{x}}(x+l) - \sqrt{x}}{(x+l)^2} \quad (=0)$

$\rightarrow \frac{x+l}{2\sqrt{x}} - \sqrt{x} = 0$

$\frac{x+l}{2} = x$

$l = x \quad \Rightarrow \quad x(I_{max}) = 5\ cm$

---

8. Homogeno magnetno polje z gostoto 0.5 T ima navpično smer. V polju se v vodoravni ravnini vrti polmetrska palica s frekvenco 2 Hz tako, da kroži prvo krajišče po krogu s polmerom 25 cm in drugo po krogu s polmerom 75 cm. Kolikšna napetost se inducira med krajiščema palice?
[$U_i = 1.57\ V$]

$B = 0'5\ T$

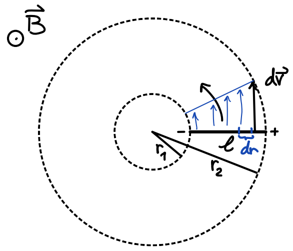
$\nu = 2\ Hz$

$r_1 = 25\ cm$

$r_2 = 75\ cm$

$l = 50\ cm$

$U_i = ?$



$U_i = \vec{v} \cdot (\vec{l} \times \vec{B})$ $\qquad \omega = 2\pi\nu = 4\pi\ s^{-1}$

$U_i = \int dU_i = \int \vec{v} \cdot (d\vec{r} \times \vec{B}) = \int_{r_1}^{r_2} vB\,dr = B\omega \int_{r_1}^{r_2} r\,dr =$ $\quad (v = \omega \cdot r)$

$= \frac{1}{2}B\omega(r_2^2 - r_1^2) = \dots = 1'57\ V$ $\qquad (T = \frac{VA}{m^2})$

---

9. Obroč iz kovine s specifičnim uporom 0.05 $\Omega mm^2/m$ ima polmer 5 cm in presek 1 mm^2. Obroč vrtimo okoli premera, ki je pravokoten na magnetno polje z gostoto 0.7 T. kolikšen povprečni navor je potreben, da se obroč zavrti desetkrat v sekundi? [$\overline{M} = 0.06\ Nm$]

$\xi = 0'05\ \frac{\Omega mm^2}{m}$

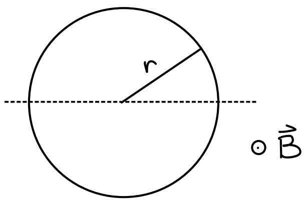
$r = 5\ cm$

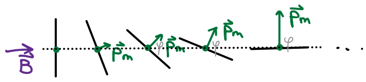
$S_0 = 1\ mm^2$

$B = 0'7\ T$

$\nu = 10\ Hz$

$\overline{M} = ?$





$\vec{M} = \vec{p}_m \times \vec{B}$

$M = p_m B \sin\varphi$ $\qquad p_m = I_i S \quad (S = \pi r^2)$

$\rightarrow I_i(\varphi) = \frac{U_i(\varphi)}{R}$

$U_i = \frac{d\Phi_m}{dt} = \frac{d(\vec{B}\vec{S})}{dt} = BS\frac{d(\cos\varphi)}{dt} = -BS\omega \sin(\omega t)$ $\quad (\varphi = \omega t)$

$\rightarrow I_i(t) = \frac{BS}{R}\omega \sin(\omega t)$

$\rightarrow M = \frac{B^2S^2}{R}\omega \sin^2(\omega t) \rightarrow \overline{M} = \frac{B^2\pi^2 r^4}{2R}\omega = \frac{B^2 \pi r^3 S_0}{4\xi} = \dots = 0'06\ Nm$

$\overline{\sin^2(\omega t)} = \frac{1}{2}$ $\qquad R = \xi \frac{2\pi r}{S_0}$

---

10. V ravnini ravnega vodnika, po katerem teče tok 300 A, je ovoj v obliki pravokotnika s stranicama 5 cm in 10 cm. Daljši stranici sta vzporedni z vodnikom, bližnja je oddaljena od vodnika za 3 cm. Izključimo tok po vodniku. Kolikšen tokovni sunek izmeri balistični galvanometer z uporom 10 Ω, ki vključen v ovoj?
[$\int I_i\, dt = 5.9 \times 10^{-7}\ As$]

$I = 300\ A$

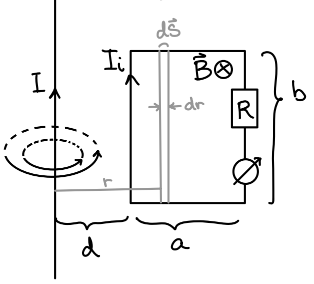
$a = 5\ cm$

$b = 10\ cm$

$d = 3\ cm$

$R = 10\ \Omega$

$e = \int I_i dt = ?$



$\Phi_m = \vec{B} \cdot \vec{S} = BS\cos\varphi$ (homogeno polje) $\qquad \Phi_m = \int \vec{B}\,d\vec{S}$ (nehomogeno)

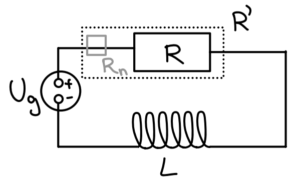
$U_i = \frac{d\Phi_m}{dt} \rightarrow \int U_i dt = \int d\Phi_m = \Phi_m = \int \vec{B}d\vec{S} = \int Bb\,dr = b\int_d^{d+a} \frac{\mu_0 I}{2\pi r}dr =$

$U_i = R \cdot I_i \int dt$ $\qquad = \frac{b\mu_0 I}{2\pi}\log\left(1 + \frac{a}{d}\right)$

$\int U_i dt = \int R I_i dt = Re \rightarrow e = \frac{1}{R}\int U_i dt = \dots = 5'88 \cdot 10^{-7}\ As$

11. Na baterijo z gonilno napetostjo $6\ V$ in uporom $2\ \Omega$ priključimo zaporedno upornik za $10\ \Omega$ in tuljavo z induktivnostjo $2 \times 10^{-3}\ H$. Kolikšni sta napetosti na tuljavi in na uporniku $10^{-4}\ s$ po vključitvi? Skicirajte časovni potek napetosti na tuljavi!
$[U_L = 3.3\ V;\ U_R = 2.3\ V]$

$U_g = 6V$
$R = 10\,\Omega$
$L = 2 \cdot 10^{-3}\ H$
$R_n = 2\,\Omega$
$t = 10^{-4}\ s$
$U_L, U_R = ?$



$R' = R + R_n = 12\,\Omega$

$U_g - U_{R'} - U_L = 0$
$U_g - R' I(t) - L\frac{dI}{dt} = 0$

$\rightarrow dt = \frac{L\,dI}{U_g - R'I}$

$\int_0^t dt = L \int_{I(0)}^{I(t)} \frac{dI}{U_g - R'I}$

$t = -\frac{L}{R'} \log\left(\frac{U_g - R'I(t)}{U_g}\right)$

$\exp\left\{-\frac{tR'}{L}\right\} = 1 - \frac{R'}{U_g} I(t)$

$\rightarrow I(t) = \frac{U_g}{R'}\left[1 - \exp\left\{-\frac{tR'}{L}\right\}\right]$

$U_L = L\frac{dI}{dt} = L\,\frac{U_g}{R'} \cdot \frac{R'}{L} \exp\left\{-\frac{tR'}{L}\right\} =$

$= U_g \exp\left\{-\frac{tR'}{L}\right\} = \dots = 3'3\ V$

$U_R = R \cdot I(t) = 10\,\Omega \cdot 0'225\ A = 2'25\ V$

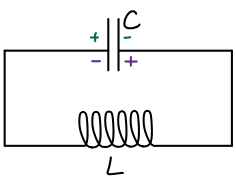
Induktivnost dolge ozke ravne tuljave: $\Phi_m = N \cdot B \cdot S = N \cdot \frac{\mu_0 N I}{l} S$

$\rightarrow L = \mu_0 \frac{N^2 S}{l}$

---

12. Električni nihajni krog sestavljata kondenzator s kapaciteto $1\ \mu F$ in tuljava z induktivnostjo $10^{-3}\ H$. Kondenzator nabijemo in nato krog s stikalom sklenemo. Kolikšna je lastna frekvenca takega nihajnega kroga? Po kolikšnem času odda kondenzator polovico svoje začetne energije tuljavi?
$[\omega_0 = 3.2 \times 10^4\ s^{-1};\ t = 2.5 \times 10^{-5}\ s]$

$C = 1\ \mu F$
$L = 10^{-3}\ H$
$\nu_0, t = ?$



← ELEKTRIČNI NIHAJNI KROG

$U_c + U_L = 0$
$\frac{e}{C} + L\frac{dI}{dt} = 0 \qquad I = \frac{de}{dt}$

$\rightarrow \frac{e}{C} + \frac{d^2e}{dt^2} L = 0$
$\rightarrow \frac{1}{LC} e + \ddot{e} = 0 \quad ; \ \omega^2 = \frac{1}{LC}$
$\omega = 2\pi\nu_0$
$\rightarrow \nu_0 = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \dots = 31'6\ kHz$

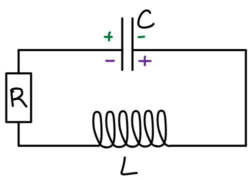
$W_c(t_{1/2}) = \frac{1}{2} W_c(t=0)$

$W_c(t) = \frac{1}{2} C U_c^2(t)$

$e(t) = e(0) \cdot \cos(\omega t)$
$e(t) = U_c(t=0) C \cdot \cos(\omega t) \quad \rightarrow U_c(t) = \frac{e(t)}{C} = U_c(t=0)\cos(\omega t)$

$\rightarrow W(t_{1/2}) = \frac{1}{2} C U_c^2(t_{1/2}) = \frac{1}{2} C U_c^2(t=0)\cos^2(\omega t) = \frac{1}{4} C U_c^2(t=0)$

$\rightarrow \cos^2(\omega t_{1/2}) = \frac{1}{2}$
$\omega t_{1/2} = \frac{\pi}{4} \quad \rightarrow t_{1/2} = \frac{\pi}{4\omega} = \dots = 24'8\ \mu s$



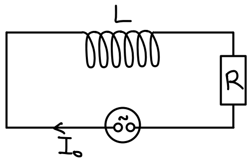
$U_c + RI + U_L = 0$

$\rightarrow \ddot{e} + 2\beta\dot{e} + \omega^2 e = 0 \quad ; \ \beta = \frac{R}{2L}$
$\omega^2 = \frac{1}{LC}$

$W_L = A_e = \int dA_e = \int U i\,de = \int_0^I L\frac{dI}{dt} I\,dt = \frac{1}{2} L I^2$

13. Tuljavo z induktivnostjo 1 H vežemo zaporedno z upornikom z uporom 3 $k\Omega$ in ju priključimo na izmenično napetost s frekvenco 50 Hz. Določite impedanco vezja! Kolikšen je fazni zamik napetosti glede na tok skozi vezje?
[$Z = 3\ K\Omega;\ \delta = -6°$]

$R = 2 k\Omega$
$L = 1\ H$
$\nu = 50\ Hz$
$\frac{U_0}{I_0},\ \delta,\ P(t) = ?$



$U_g = U_0 \cos(\omega t)$ "vzrok"
$I = I_0 \cos(\omega t - \delta)$ "posledica"

$$U_g = U_R + U_L = IR + L\frac{dI}{dt}$$

$$U_0 \cos(\omega t) = I_0 R \cos(\omega t - \delta) + L I_0 (-\omega) \sin(\omega t - \delta)$$
$$U_0 \cos(\omega t) + 0 \cdot \sin(\omega t) = I_0 R[\cos(\omega t)\cos\delta + \sin(\omega t)\sin\delta] - I_0 \omega L \cdot [\sin(\omega t)\cos\delta - \cos(\omega t)\sin\delta]$$

$\rightarrow$ $\cos(\omega t):\ U_0 = I_0 R\cos\delta + I_0\omega L \sin\delta$
$\sin(\omega t):\ 0 = I_0 R \sin\delta - I_0 \omega L \cos\delta$ $\rightarrow \tan\delta = \frac{\omega L}{R}$ $\rightarrow \delta = 6°$

$$\frac{U_0}{I_0} = R\cos\delta + \omega L \sin\delta = R\cos\delta\left(1 + \frac{\omega L}{R}\tan\delta\right) = R\sqrt{1 + \frac{\omega^2 L^2}{R^2}}^{\,-1}\left(1 + \frac{\omega^2 L^2}{R^2}\right) = R\sqrt{1 + \frac{\omega^2 L^2}{R^2}} =$$
$$= \sqrt{R^2 + \omega^2 L^2} = \cdots = 3{,}016\ k\Omega = Z$$
$\uparrow$ IMPEDANCA

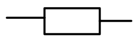
$$P(t) = U(t)I(t) = U_0\cos(\omega t)\cdot I_0\cos(\omega t - \delta) = U_0 I_0 \cos(\omega t)(\cos(\omega t)\cos\delta + \sin(\omega t)\sin\delta)$$

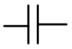
$$\overline{P(t)} = U_0 I_0 (\underbrace{\cos^2\omega t}_{\frac{1}{2}} \cdot \cos\delta + \underbrace{\cos\omega t \cdot \sin\omega t}_{0} \cdot \sin\delta) = \frac{U_0 I_0}{2}\cos\delta$$

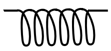
2. način za impedanco:
$U_g = U_0\cos(\omega t)$ $\rightarrow U_0 e^{i\omega t}$
$I = I_0 \cos(\omega t - \delta)$ $\rightarrow I_0 e^{i(\omega t - \delta)}$

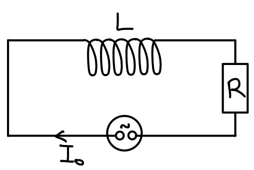
$$\rightarrow Z = \frac{U_g}{I} = \frac{U_0 \exp\{i\omega t\}}{I_0 \exp\{i(\omega t - \delta)\}} = \underbrace{\frac{U_0}{I_0}}_{|Z|} e^{i\delta} \in \mathbb{C}$$

3 posebni elementi:

: $U_g = RI \rightarrow Z_R = \frac{U_g}{I} = R$ ($\delta = 0$)

: $U_g = \frac{e}{C}\ /\frac{d}{dt} \rightarrow i\omega U_g = \frac{I}{C} \rightarrow Z_C = \frac{U_g}{I} = \frac{1}{i\omega C}$ ($\delta = -\frac{\pi}{2}$)

: $U_g = L\frac{dI}{dt} = Li\omega I \rightarrow Z_L = \frac{U_g}{I} = i\omega L$ ($\delta = \frac{\pi}{2}$)



$\rightarrow Z = Z_R + Z_L = R + i\omega L$

$|Z| = \sqrt{R^2 + \omega^2 L^2}$
$\tan\delta = \frac{\omega L}{R}$
} enako kot prej