Fizika 2 (PREDAVANJA)

UVOD

SILE

Kvalitativno: fizikalne količine, ki podajajo vpliv okolice na opazovano telo oz. sistem

Kvantitativno: temelj na 5 pravilih (definicije, predpostavke /fizikalni zakoni/)

$\vec{F}$ ... skupna sila okolice na opazovano telo
m ... masa opazovanega telesa
$\vec{v}$ ... hitrost opazovanega telesa
$\vec{a}$ ... pospešek opazovanega telesa
d_{min} ... minimalna razdalja opazovanega telesa od okoliških teles

(1) (predpostavka): $\boxed{\lim_{d_{min}\to\infty} \vec{F} = \vec{0}}$

(2) (definicija inercialnega opazovalnega /koordinatnega/ sistema)

$$\boxed{d_{min}\to\infty \overset{(1)}{\Rightarrow} \vec{F}=\vec{0}: \quad S: \ \vec{v}=\text{konst.} \overset{(\text{def})}{\Rightarrow} S \text{ inercialen}}$$

sistem

Komentar: 1. Newtnov zakon v smislu zgornje definicije.

(3) (definicija $\vec{F}$): S inercialen, $\vec{a}, m$ $\boxed{\vec{F} \equiv m\cdot\vec{a}}$

Komentar: 2. Newtonov zakon ; $\vec{a}=0 \Rightarrow \vec{F}=0$
(1. Newtonov zakon)

Komentar: daljnosežne implikacije

(4) (zakon - načelo superpozicije)

$\vec{F}_i$ ... sila i-tega okoliškega telesa na opazovano telo

(če so vsa ostala okoliška telesa daleč stran $\Rightarrow \vec{F}=\vec{F}_i$)

$$\boxed{\vec{F}=\sum_i \vec{F}_i}$$

Komentar: Newton v 'Principih' (Axioms or laws of motion), Posledica 1 - ta zakon samo posledica prvih dveh zakonov

(5) (definicija): dve telesi: $\vec{F}_{1\to2}$ in $\vec{F}_{2\to1}$

Če velja: $\vec{F}_{2\to1} = -\vec{F}_{1\to2}$, $\overset{(\text{def.})}{\text{potem}}$ za sili $\vec{F}_{1\to2}$ in $\vec{F}_{2\to1}$ velja 3. Newtonov zakon.

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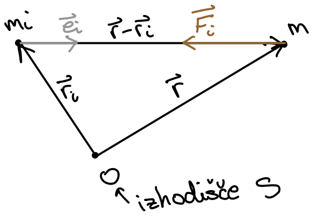
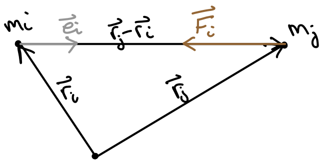


$$\hat{e}_i = \frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|}$$

$$\vec{F}_i = -\frac{\kappa m_i m}{|r-r_i|^2}\hat{e}_i = -\frac{\kappa m_i m}{|\vec{r}-\vec{r}_i|^3}(\vec{r}-\vec{r}_i)$$

$$|\vec{r}-\vec{r}_i| \to \infty : \lim_{|\vec{r}-\vec{r}_i|\to\infty} \vec{F}_i = 0$$



$$\vec{F}_{i\to j} = -\frac{\kappa m_i m_j}{|\vec{r}_j-\vec{r}_i|^3}(\vec{r}_j-\vec{r}_i) \qquad |r_i-r_j|^3 = |\vec{r}_j-\vec{r}_i|^3 \wedge (\vec{r}_i-\vec{r}_j) = -(\vec{r}_j-\vec{r}_i)$$

$$\vec{F}_{j\to i} = -\frac{\kappa m_i m_j}{|\vec{r}_i-\vec{r}_j|^3}(\vec{r}_i-\vec{r}_j) = \frac{\kappa m_i m_j}{|\vec{r}_j-\vec{r}_i|^3}(\vec{r}_j-\vec{r}_i) = -\vec{F}_{i\to j}$$

(velja 3. Newtonov zakon)

GIBALNA KOLIČINA

$\vec{F}_1$ ... skupna sila na 1. telo
m_1 ... masa 1. telesa
$\vec{v}_1$ ... hitrost 1. telesa
$\int_{t_1}^{t_2} \vec{F}_1\, dt$ ... sunek vseh sil na 1. telo (t_1, t_2 začetni in končni čas)

$G_1 = m_1 \cdot \vec{v}_1$ ... gibalna količina 1. telesa

$$\Delta\vec{G}_1 = \vec{G}_{1K} - \vec{G}_{1Z}$$

$$\boxed{\text{IZREK: } \int_{t_1}^{t_2} \vec{F}_i\, dt = \Delta\vec{G}_i}$$

$\Delta\vec{G}_i = \vec{G}_{i,2} - \vec{G}_{i,1}$; $\vec{G}_{i,1} = m_i \cdot \vec{v}_{i,1}$, $\vec{G}_{i,2} = m_i \cdot \vec{v}_{i,2}$ $\qquad m_i = \text{konst.} \,/\, m \neq m(\vec{v})$

Dokaz:
$$\int_{t_1}^{t_2} \vec{F}_i\, dt \overset{(3)}{=} \int_{t_1}^{t_2} m_i \vec{a}_i\, dt = \int_{t_1}^{t_2} m_i \dot{\vec{v}}_i\, dt = \int_{t_1}^{t_2} (m_i\dot{\vec{v}_i})\, dt = m_i\vec{v}_i\Big|_{t_1}^{t_2} = m_i\vec{v}_{i,2} - m_i\vec{v}_{i,1} =$$
$$= \Delta\vec{G}_i \qquad \square$$

SISTEM 2 TELES:

$\vec{G} = \vec{G}_1 + \vec{G}_2$; $\quad \vec{G}_Z = \vec{G}_{1,Z} + \vec{G}_{2,Z}$, $\quad \vec{G}_K = \vec{G}_{1,K} + \vec{G}_{2,K}$

$\vec{F}_{2\to1}$, $\vec{F}_{1\to2}$ ... "notranji" sili
$\vec{F}_1$, $\vec{F}_2$ ... skupni sili

$$\left.\begin{aligned} \vec{F}_{1,Z} &= \vec{F}_1 - \vec{F}_{2\to1} \\ \vec{F}_{2,Z} &= \vec{F}_2 - \vec{F}_{1\to2} \end{aligned}\right\} \text{"zunanji" sili}$$

$$\int_{t_a}^{t_b} \vec{F}_1\, dt = \vec{G}_{1,k} - \vec{G}_{1,z}$$

$$\int_{t_a}^{t_b} (\vec{F}_{1,z} + \vec{F}_{2\to 1})\, dt = \int_{t_a}^{t_b} \vec{F}_{1,z}\, dt + \int_{t_a}^{t_b} \vec{F}_{2\to 1}\, dt = \vec{G}_{1,k} + \vec{G}_{1,z}$$

$$\int_{t_a}^{t_b} \vec{F}_{2,z}\, dt + \int_{t_a}^{t_b} \vec{F}_{1\to 2}\, dt = \vec{G}_{2,k} - \vec{G}_{2,z}$$

<u>Predpostavka: $\vec{F}_{2\to 1} = -\vec{F}_{1\to 2}$</u> ← velja 3. Newtonov zakon

$$\int_{t_a}^{t_b} \vec{F}_{1,z}\, dt + \int_{t_a}^{t_b} \vec{F}_{2,z}\, dt + \int_{t_a}^{t_b} \vec{F}_{2\to 1}\, dt + \int_{t_a}^{t_b} \vec{F}_{1\to 2}\, dt =$$

$$= \int_{t_a}^{t_b} (\vec{F}_{1,z} + \vec{F}_{2,z})\, dt + \int_{t_a}^{t_b} \underbrace{(\vec{F}_{2\to 1} + \vec{F}_{1\to 2})}_{= 0}\, dt = \int_{t_a}^{t_b} \vec{F}_z\, dt \quad ; \quad \vec{F}_z = \vec{F}_{1,z} + \vec{F}_{2,z}$$

$\vec{G}_{1,k} + \vec{G}_{2,k} - \vec{G}_{1,z} - \vec{G}_{2,z} = \vec{G}_k - \vec{G}_z = \Delta\vec{G}$ (razlika skupne gibalne količine)

$$\Rightarrow \boxed{\int_{t_a}^{t_b} \vec{F}_z\, dt = \Delta\vec{G}}$$

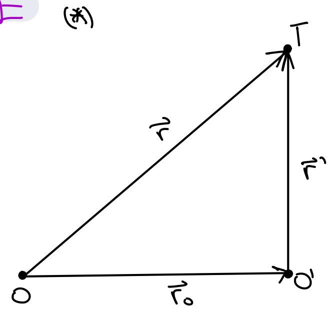
Primer: $\int_{t_a}^{t_b} \vec{F}_z = 0$ ← sistem izoliran od okolice

$\Rightarrow \Delta\vec{G} = 0$ ali $\vec{G}_k = \vec{G}_z$ ← gibalna količina se ohranja

GALILEJEVE TRANSFORMACIJE (*)

sistem S, izhodišče O \
sistem S', izhodišče O' $\Big\}\ \vec{r}_0$

$\vec{r}_0 = \vec{v}_0 \cdot t$; $\vec{v}_0 = \text{konst.}$



$\vec{r} = \vec{r}_0 + \vec{r}'$

$\Rightarrow \vec{r}' = \vec{r} - \vec{r}_0 = \vec{r} - \vec{v}_0 t$

$$\vec{v} \equiv \frac{d\vec{r}}{dt}$$

Predpostavka: <u>$t = t'$</u>

$$\vec{v}' \equiv \frac{d\vec{r}'}{dt'} = \frac{d\vec{r}'}{dt} = \frac{d}{dt}(\vec{r} - \vec{v}_0 t) = \frac{d\vec{r}}{dt} - \frac{d}{dt}(\vec{v}_0 t) = \vec{v} - \vec{v}_0$$

$$\vec{a} \equiv \frac{d\vec{v}}{dt}$$

$$\vec{a}' \equiv \frac{d\vec{v}'}{dt'} = \frac{d\vec{v}'}{dt} = \frac{d}{dt}(\vec{v} - \vec{v}_0) = \frac{d\vec{v}}{dt} = \vec{a}$$

(1) S inercialen sistem $\Rightarrow$ S' inercialen sistem

$$\vec{F}=0 \Rightarrow \vec{a}=0 \Rightarrow \vec{a}'=0$$

(2) $\vec{F}'$ skupna sila na telo v S': $\vec{F}' = m'\cdot\vec{a}'$; $m'=m$ (predpostavka)

$$= m\cdot\vec{a} \qquad \vec{a}'=\vec{a} \ (*)$$

$$= \vec{F} \leftarrow \text{sile invariantne}$$

MATRIČNI ZAPIS GALILEJEVIH TRANSFORMACIJ:

$$\hat{e}_x = \hat{e}_x' \ ; \quad \hat{e}_y = \hat{e}_y' \ ; \quad \hat{e}_z = \hat{e}_z'$$

$$\vec{v}_0 = v_0 \cdot \hat{e}_x$$

$$\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \vec{r}' = x'\hat{e}_x' + y'\hat{e}_y' + z'\hat{e}_z' = x'\hat{e}_x + y'\hat{e}_y + z'\hat{e}_z = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\vec{r}' = \vec{r} - \vec{v}_0 t = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x - v_0 t \\ y \\ z \end{bmatrix}$$

$${}^{(4)}x \equiv \begin{bmatrix} c_0 t \\ x \\ y \\ z \end{bmatrix} \Rightarrow {}^{(4)}x' = \begin{bmatrix} c_0 t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} c_0 t \\ x - v_0 t \\ y \\ z \end{bmatrix}$$

4-dimenzije

c_0 - hitrost svetlobe v vakuumu

$$(G \neq \vec{G}) \qquad G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta_0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \ ; \ \beta_0 = \frac{v_0}{c_0} \leftarrow \text{matrika Galilejeve transformacije}$$

$$\Rightarrow \boxed{{}^{(4)}x' = G\,{}^{(4)}x}$$

$$S_0 \to S_1: \ v_1, \ \beta_1 = \frac{v_1}{c_0} \ : \quad G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_1 \to S_2: \ v_2; \ \beta_2 = \frac{v_2}{c_0} \ : \quad G_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta_2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

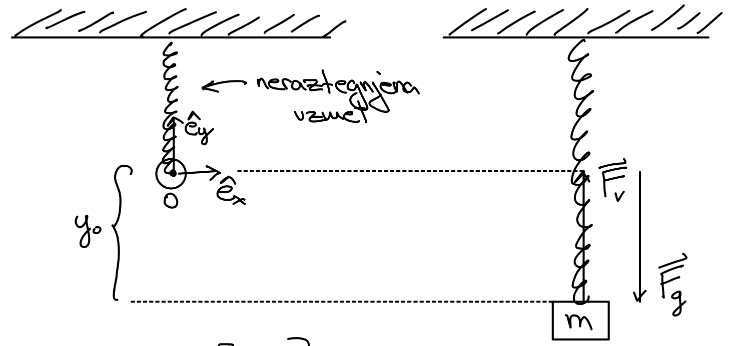
$$S_0 \to S_1 \to S_2: \quad G_2 \cdot G_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\beta_1\beta_2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Leftarrow S_0 \to S_2$$

1. MEHANSKO NIHANJE in VALOVANJE

ENOSTAVNA NIHALA – ENAČBA (LASTNEGA DUŠENEGA) NIHANJA

UTEŽ NA VIJAČNI VZMETI

(a)



$$\vec{F}_g = \begin{bmatrix} 0 \\ -mg_0 \\ 0 \end{bmatrix} \qquad \vec{F}_v = \begin{bmatrix} 0 \\ -ky_0 \\ 0 \end{bmatrix} \; ; \quad \left.\begin{matrix} y_0 < 0 \\ k > 0 \end{matrix}\right\} \Rightarrow -ky_0 > 0 \Rightarrow \vec{F}_v \uparrow$$

$g_0 \sim 10 \frac{m}{s^2}$ (Hookov zakon)

$$\Rightarrow \boxed{mg_0 = -ky_0}$$

Točki (3) in (4): (3): $\vec{a} = 0 \Rightarrow \vec{F} = 0$

(4): $\vec{F} = \vec{F}_g + \vec{F}_v = -mg_0\,\hat{e}_y - ky_0\,\hat{e}_y = -(mg_0 + ky_0)\,\hat{e}_y = 0$

(b) v smeri $\hat{e}_y$: $y_0 \to y$

po (3): $\vec{F} = m \cdot \vec{a} \equiv m \cdot \dot{\vec{v}} = m \cdot \ddot{\vec{r}}$; $\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \ddot{\vec{r}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \ddot{x}\,\hat{e}_x + \ddot{y}\,\hat{e}_y + \ddot{z}\,\hat{e}_z$

po (4): $\vec{F} = \vec{F}_g + \vec{F}_v + \vec{F}_u$ ← sila upora

$$\vec{F}_g = -mg_0\hat{e}_y \; ; \; \vec{F}_v = -ky_0\,\hat{e}_y \; ; \; \vec{F}_u = -C\vec{v} \; ; \; C > 0 \; \to \; \vec{v} = -C\vec{v}\,\hat{e}_y = -C\dot{y}\,\hat{e}_y$$

↑ LINEARNI ZAKON UPORA

$$(-mg_0 - ky_0 - C\dot{y})\,\hat{e}_y = m\ddot{x}\,\hat{e}_x + m\ddot{y}\,\hat{e}_y + m\ddot{z}\,\hat{e}_z$$

$$\Rightarrow m\ddot{x}\,\hat{e}_x + (mg_0 + ky_0 + C\dot{y} + m\ddot{y})\,\hat{e}_y + m\ddot{z}\,\hat{e}_z = 0$$

$$\Rightarrow \begin{cases} m\ddot{x} = 0 \\ m\ddot{y} + mg_0 + ky + C\dot{y} = 0 \\ m\ddot{z} = 0 \end{cases}$$

$\Rightarrow \dot{x} = \text{konst.}$

$v_x(t=0) = 0 \Rightarrow \dot{x} = 0 \Rightarrow x = \text{konst.}$

$\Rightarrow \underline{x = 0}$

$\Rightarrow z = \text{konst.}$

$\Rightarrow \underline{z = 0}$

$$m\ddot{y} + C\dot{y} + ky - \overbrace{ky_0}^{mg_0} = 0 \quad /: m$$

$$\Rightarrow \ddot{y} + \beta\dot{y} + \omega_0^2(y - y_0) = 0 \; ; \quad \boxed{\beta = \frac{C}{m}} > 0 \quad [\beta] = s^{-1}$$

↖ KOEFICIENT DUŠENJA

$$\boxed{\omega_0^2 = \frac{k}{m}} \quad [\omega_0^2] = s^{-2}$$

$y' = y - y_0$

$\dot{y}' = \dot{y}$, $\ddot{y}' = \ddot{y}$ $\Rightarrow$ $y = y' + y_0$

$$\Rightarrow \boxed{\ddot{y}' + \beta\dot{y}' + \omega_0^2 y' = 0} \leftarrow \text{ENAČBA (DUŠENEGA LASTNEGA) NIHANJA}$$

(homogena linearna diferencialna enačba 2. reda s konstantnimi koeficienti)

Reševanje enačbe $\ddot{y}' + \beta\dot{y}' + \omega_0^2 y' = 0$:

$y' = Ae^{\lambda t}$; A, λ konst.

$\Rightarrow \dot{y}' = \lambda y'$ in $\ddot{y}' = \lambda^2 y'$

$\Rightarrow (\lambda^2 + \beta\lambda + \omega_0^2) Ae^{\lambda t} = 0$ za $\forall t$

(1) $A = 0 \Rightarrow y' = 0$ ← TRIVIALNA REŠITEV

(2) Rešujemo torej kvadratno enačbo: $\lambda^2 + \beta\lambda + \omega_0^2 = 0$.

$D = \beta^2 - 4\omega_0^2 \equiv -4\omega^2$

$\Rightarrow \omega^2 = \omega_0^2 - \frac{\beta^2}{4}$

Dobimo več možnosti:

(a) $D < 0$ $(\omega^2 > 0)$: PODKRITIČNO DUŠENJE

$\sqrt{D} = \sqrt{-4\omega^2} = 2i\omega$; $\omega = +\sqrt{\omega^2}$

$\lambda_{1,2} = \frac{-\beta \pm 2i\omega}{2} = -\frac{\beta}{2} \pm i\omega$

$\Rightarrow y_1' = A_1 \exp(-\frac{\beta}{2}t + i\omega t) = A_1 \exp(-\frac{\beta}{2}t) \exp(i\omega t)$
in
$y_2' = A_2 \exp(-\frac{\beta}{2}t) \exp(-i\omega t)$

$\ddot{y}_1' + \beta\dot{y}_1' + \omega_0^2 y_1' = 0$ in $\ddot{y}_2' + \beta\dot{y}_2' + \omega_0^2 y_2' = 0$

Enačba linearna $\Rightarrow y' = y_1' + y_2'$ tudi rešitev

$(\ddot{y_1' + y_2'}) + \beta(\dot{y_1' + y_2'}) + \omega_0^2(y_1' + y_2') = \ddot{y}_1' + \ddot{y}_2' + \beta\dot{y}_1' + \beta\dot{y}_2' + \omega_0^2 y_1' + \omega_0^2 y_2' = 0$

$\Rightarrow \boxed{y' = \exp(-\frac{\beta}{2}t)[A_1 \exp(i\omega t) + A_2 \exp(-i\omega t)]}$

Uporabimo Eulerjevo formulo: $\exp(\pm i\omega t) = \cos(\omega t) \pm i\sin(\omega t)$

$\Rightarrow y' = \exp(-\frac{\beta}{2}t)[(A_1 + A_2)\cos(\omega t) + i(A_1 - A_2)\sin(\omega t)] =$

$= \exp(-\frac{\beta}{2}t)[B_1 \cos(\omega t) + B_2 \sin(\omega t)]$

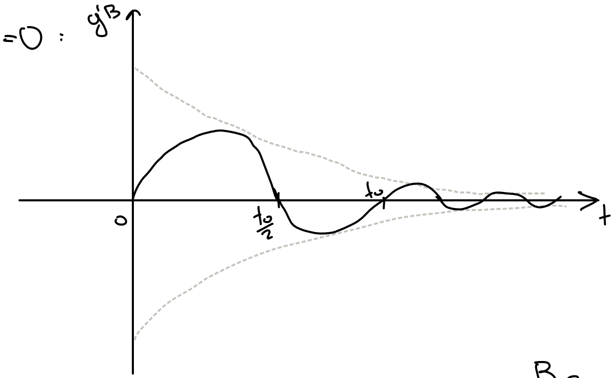
$= \boxed{B\exp(-\frac{\beta}{2}t)\sin(\omega t + \delta)}$ „fazni premik"

$\sin(\omega t + \delta) = \cos\delta \sin(\omega t) + \sin\delta \cos(\omega t)$

$\Rightarrow B\exp(-\frac{\beta}{2}t)\sin(\omega t + \delta) = \exp(-\frac{\beta}{2}t)[B\cos\delta \sin(\omega t) + B\sin\delta \cos(\omega t)]$

$\Rightarrow B_1 = B\sin\delta$, $B_2 = B\cos\delta$, $\tan\delta = \frac{B_1}{B_2}$ $\rightarrow B = +\sqrt{B_1^2 + B_2^2}$

Primer: $\delta = 0$:



t_0 = lastni nihajni čas

$\omega t_0 = 2\pi \Rightarrow \boxed{t_0 = \frac{2\pi}{\omega}}$

$\nu = \frac{1}{t_0} \Rightarrow \omega = 2\pi\nu$

$B\exp(-\frac{\beta}{2}t) \leftarrow$ DUŠENJE

B in δ iz "začetnih" pogojev:

$y'(t=0)$ in $\dot{y}'(t=0)$

$$y'(t) = B\exp(-\tfrac{\beta}{2}t)\sin(\omega t+\delta) \Rightarrow \boxed{y'(t=0) = B\sin\delta}$$

$$\dot{y}'(t) = -\tfrac{\beta}{2}B\exp(-\tfrac{\beta}{2}t)\sin(\omega t+\delta) + B\omega\exp(-\tfrac{\beta}{2}t)\cos(\omega t+\delta)$$

$$\Rightarrow \boxed{\dot{y}'(t=0) = -\tfrac{\beta}{2}B\sin\delta + B\omega\cos\delta}$$

ENERGIJA NIHALA

$$W_k = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(\dot{y}')^2 \longrightarrow v = \dot{y} = \dot{y}' \quad (y \neq y')$$

$$W_{pr} = \tfrac{1}{2}ky^2 = \tfrac{1}{2}k(y'+y_0)^2$$

$$W_p = mg_0y = mg_0(y'+y_0)$$

$$\Rightarrow W = W_k + W_p + W_{pr}$$

Primer: nedušeno nihanje: $\beta \to 0$

$$\omega^2 = \omega_0^2 - \frac{\beta^2}{4} = \omega_0^2 = \frac{k}{m}$$

$$\Rightarrow y' = B\exp(-\tfrac{\beta}{2}t)\sin(\omega t+\delta) = B\sin(\omega_0 t+\delta)$$

$$\Rightarrow \dot{y}' = B\omega_0\cos(\omega_0 t+\delta)$$

$$W_k = \tfrac{1}{2}mB^2\omega_0^2\cos^2(\omega_0 t+\delta) = \tfrac{1}{2}mB^2\tfrac{k}{m}\cos^2(\omega_0 t+\delta) = \tfrac{kB^2}{2}\cos^2(\omega_0 t+\delta)$$

$$W_{pr} = \tfrac{1}{2}kB^2\sin(\omega_0 t+\delta) + ky_0B\sin(\omega_0 t+\delta) + \tfrac{1}{2}ky_0^2$$

$$W_p = mg_0y_0 + mg_0B\sin(\omega_0 t+\delta)$$

$$\Rightarrow W = \tfrac{1}{2}kB^2 + \underbrace{(ky_0+mg_0)}_{=0}B\sin(\omega_0 t+\delta) + \tfrac{1}{2}ky_0^2 + mg_0y_0$$

$$\Rightarrow W = \tfrac{1}{2}kB^2 + \tfrac{1}{2}ky_0^2 + mg_0y_0 = \tfrac{1}{2}kB^2 - \tfrac{1}{2}ky_0^2 = \text{konst.}$$

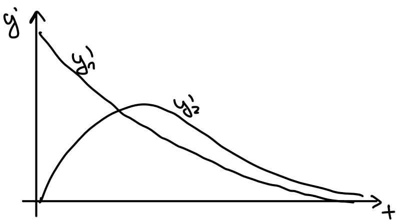
Dušenje: $B \to B\exp(-\frac{\beta}{2}t)$

(b) $D=0$ $(\omega=0)$: KRITIČNO DUŠENJE

$$\lambda = -\frac{\beta}{2} \Rightarrow \boxed{y_1' = B_1\exp(-\tfrac{\beta}{2}t)}$$

Trdimo: $y_2' = B_2t\exp(-\frac{\beta}{2}t)$ tudi rešitev

$\Rightarrow y' = (B_1 + B_2 t)\exp(-\frac{\beta}{2}t)$



(c) $D > 0$ $(\omega^2 < 0)$: NADKRITIČNO DUŠENJE

$$D = -4\omega^2 > 0 \Rightarrow \omega^2 < 0 \Rightarrow \omega = \pm i|\omega|$$

$$D = 4|\omega|^2 \Rightarrow \lambda_{1,2} = -\frac{\beta}{2} \pm |\omega|$$

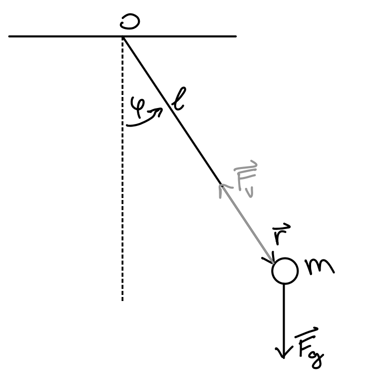
$$\Rightarrow y' = B_1 \exp(\lambda_1 t) + B_2 \exp(\lambda_2 t) \quad ; \quad \begin{array}{l} \lambda_2 = -\frac{\beta}{2} - |\omega| < 0 \ ✓ \\ \lambda_1 = -\frac{\beta}{2} + |\omega| < 0 \ ✓ \end{array}$$

$$\omega^2 = \omega_0^2 - \frac{\beta^2}{4} < 0 \rightarrow -\omega^2 = \frac{\beta^2}{4} - \omega_0^2 = |\omega|^2 \rightarrow |\omega| = \frac{\beta}{2}\underbrace{\sqrt{1 - \frac{4\omega_0^2}{\beta^2}}}_{<1} < \frac{\beta}{2}$$

Zelo močno dušenje: $\frac{4\omega_0^2}{\beta^2} \ll 1 \Rightarrow |\omega| \doteq \frac{\beta}{2}\left(1 - \frac{2\omega_0^2}{\beta^2}\right)$

$$\lambda_1 = \underbrace{-\frac{\beta}{2} + \frac{\beta}{2}}_{0} - \underbrace{\frac{\omega_0^2}{\beta^2}}_{\ll 1}$$

NITNO ali MATEMATIČNO NIHALO (nedušeno)



$$\boxed{\vec{M} = \vec{r} \times \vec{F}_g + \underbrace{\vec{r} \times \vec{F}_v}_{\overset{!}{=}0 \leftarrow \vec{r} \parallel \vec{F}_v}} = \vec{r} \times \vec{F}_g$$

$$\vec{F}_g = -mg_0 \hat{e}_y \ ; \ \vec{r} = l\sin\varphi\, \hat{e}_x - l\cos\varphi\, \hat{e}_y$$

$$\vec{M} = \vec{r} \times \vec{F}_g = -mg_0 l \sin\varphi\, \hat{e}_x \times \hat{e}_y = -mg_0 l \sin\varphi\, \hat{e}_z$$

$$\Rightarrow \vec{M} = M_z \hat{e}_z \Rightarrow \boxed{M_z = -mg_0 l \sin\varphi}$$

Newtonov zakon za vrtenje okoli fiksne osi: $(\hat{e}_z)$

$$\boxed{M_z = J_z \alpha = J_z \ddot{\varphi}} \ ; \ J_z = ml^2$$

$$-mg_0 l \sin\varphi = ml^2 \ddot{\varphi} \Rightarrow \boxed{\ddot{\varphi} + \frac{g_0}{l}\sin\varphi = 0}$$

$\varphi \ll 1$ $([\varphi] = \text{rad}) \Rightarrow$ Taylor: $\sin\varphi \sim \varphi$

$$\Rightarrow \boxed{\ddot{\varphi} + \frac{g_0}{l}\varphi = 0} \quad \omega_0^2 = \frac{g_0}{l} \Rightarrow \boxed{t_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g_0}}}$$

enačba nihanja

1. POSKUS: t_0 nitnega nihala

$t_{10} \rightarrow t_0 = \frac{t_{10}}{10}$ $\quad t_{10} = 19,87\ \text{s} \Rightarrow t_0 = 1'99\ \text{s}$

$t_0 = 2\pi\sqrt{\frac{l}{g}}$ $\leftarrow 10\frac{m}{s^2}$ $\quad l = 1\,m \Rightarrow t_0 = 2\pi\sqrt{\frac{1}{10}s^2} \doteq 2\pi \cdot 0'33\ \text{s} \doteq 2\ \text{s}$ majhno odstopanje

2. POSKUS: t_0 nihala na vijačno vzmet:

$t_{10} = 13'24\ s \Rightarrow \underline{t_0 \approx 1'3\ s}$

$F_v = kx$

$F_g = m_1 g_0$

$t_0 = 2\pi\sqrt{\frac{m}{k}}$; $m = 400\ g = 0'4\ kg$

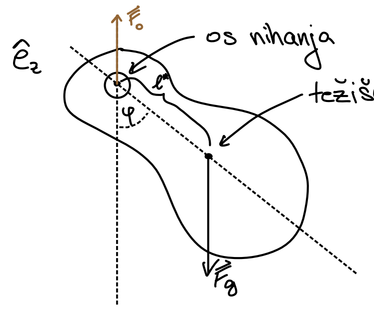
$\left.\begin{array}{l} x = 0'2\,m \\ \Delta m = 0'2\,kg \end{array}\right\} \Rightarrow k = \frac{m_1 \cdot g_0}{x}$

$t_0 = 2\pi\sqrt{\frac{2m_1 \cdot x}{m_1 g_0}} = 2\pi \cdot \sqrt{\frac{2 \cdot 0'2\ m}{10\ m}\ s^2} =$

$\doteq \underline{1'2\ s}$

FIZIČNA NIHALA

(1)



M_z: navor vseh sil okoli osi

$\vec{M}_0 = \vec{r}_0 \times \vec{F}_0 = \vec{0}$ $(\vec{r}_0 = 0)$

$M_z = -mg\ell^* \sin\varphi$ (M_g... navor sile teže)

$\boxed{M_z = J_z \alpha}$

vztrajnostni moment (odvisnost: m, oblike)

$\alpha = \ddot{\varphi}$

$\Rightarrow -mg\ell^* \sin\varphi = J_z \ddot{\varphi} \Rightarrow \boxed{\ddot{\varphi} + \frac{mg\ell^*}{J_z}\sin\varphi = 0}$

$\varphi \ll 1: \sin\varphi \doteq \varphi \Rightarrow \boxed{\ddot{\varphi} + \underbrace{\frac{mg_0\ell^*}{J_z}}_{\omega_0^2 = \frac{mg_0\ell^*}{J_z}}\varphi = 0}$

$\boxed{t_0 = \frac{2\pi}{\omega_0}}$

Primer: $\ell^* = \frac{\ell}{2}$

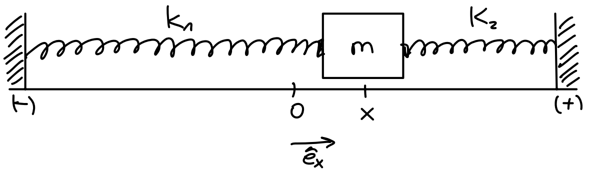
$\underline{J_z} = \frac{1}{12}m\ell^2 + m\left(\frac{\ell}{2}\right)^2 = \left(\frac{1}{12} + \frac{1}{4}\right)m\ell^2 = \underline{\frac{1}{3}m\ell^2}$

vrtenje okrog osi — premik

$\Rightarrow \omega_0^2 = \frac{3mg_0\ell}{2\,m\ell^2} = \frac{3}{2}\frac{g_0}{\ell}$

$\Rightarrow \boxed{t_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{2}{3}\frac{\ell}{g_0}}}$

(2)



a) $\ell_{1,0}$
b) $\ell_{1,r}$ $\Big\} \Rightarrow \Delta\ell_{1,r} = \ell_{1,r} - \ell_{1,0} \Rightarrow$

sila prve vzmeti

$\vec{F}_{1,r} = -k_1 \Delta\ell_{1,r}\,\hat{e}_x$

sila druge vzmeti

$\vec{F}_{2,r} = k_2 \Delta\ell_{2,r}\,\hat{e}_x$

$$\vec{F}_1 + \vec{F}_2 + \underbrace{\vec{F}_g + \vec{F}_r}_{=\vec{0}} = \vec{F}_1 + \vec{F}_2$$

V ravnovesni legi: $\vec{F}_1 = \vec{F}_{1,r}$, $\vec{F}_2 = \vec{F}_{2,r}$

$$\Rightarrow \vec{F}_{1,r} + \vec{F}_{2,r} \underset{(*)}{=} \vec{0} \quad \Rightarrow \quad (-k\Delta l_{1,r} + k\Delta l_{2,r})\,\hat{e}_x = 0$$

$$\Rightarrow \boxed{\frac{\Delta l_{1,r}}{\Delta l_{2,r}} = \frac{k_2}{k_1}}$$

c) telo v legi x:

$$\Delta l_1 = \Delta l_{1,r} + x \quad \text{in} \quad \Delta l_2 = \Delta l_{2,r} - x$$

$$\Rightarrow \vec{F}_1 = -k_1 \Delta l_1 \hat{e}_x = -k_1 \Delta l_{1,r} \hat{e}_x - k_1 x \hat{e}_x = \vec{F}_{1,r} - k_1 x \hat{e}_x$$

$$\Rightarrow \vec{F}_2 = k_2 \Delta l_2 \hat{e}_x = k_2 \Delta l_{2,r} \hat{e}_x - k_2 x \hat{e}_x = \vec{F}_{2,r} - k_2 x \hat{e}_x$$

$$\Rightarrow \vec{F}_1 + \vec{F}_2 = \vec{F}_{1,r} - k_1 x \hat{e}_x + \vec{F}_{2,r} - k_2 x \hat{e}_x \overset{(*)}{=} \boxed{-(k_1+k_2)x\hat{e}_x = m\ddot{x}\,\hat{e}_x}$$

$$\Rightarrow (m\ddot{x} + (k_1+k_2)x)\,\hat{e}_x = 0$$

$$\Rightarrow \boxed{\ddot{x} + \frac{k_1+k_2}{m} x = 0} \leftarrow \text{ENAČBA NIHANJA}$$

$$\Rightarrow \omega_0^2 = \frac{k_1+k_2}{m}$$

$$\Rightarrow \boxed{t_0 = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k_1+k_2}}}$$

VSILJENO (DUŠENO) NIHANJE

Spomnimo se: $[\underbrace{m\ddot{y} + C\dot{y} + k(y-y_0)}_{\Sigma F}]\,\hat{e}_y = 0$

$$y' = y - y_0$$

$$\Rightarrow \boxed{m\ddot{y}' + C\dot{y}' + ky' = 0}$$

Dodatna (vsiljena) sila: sinusna: $\nu_v = \omega_v = 2\pi\nu_v$

$$\Sigma \vec{F} = -(C\dot{y} + k(y-y_0))\,\hat{e}_y + \vec{F}_v \quad ; \quad \vec{F}_v = \underset{\text{amplituda vsiljenja}}{F_0} \sin(\omega_v t)\,\hat{e}_y$$

$$\boxed{m\ddot{y}' + C\dot{y}' + ky' = F_0 \sin(\omega_v t)} \leftarrow \text{ENAČBA VSILJENEGA NIHANJA}$$

(nehomogena)

$$\boxed{\ddot{y}' + \beta\dot{y}' + \omega_0^2 y' = \frac{F_0}{m}\sin(\omega_v t)}$$

$$y' = \underset{\text{rešitev homogenega dela}}{y'_h} + \underset{\text{partikularna rešitev}}{y'_p}$$

$$y'_h = B e^{(-\frac{\beta}{2}t)} \sin(\omega t + \delta) \quad \text{(podkritično nihanje)}$$

$$\frac{\beta}{2}t \gg 1 \Rightarrow y'_h \to 0$$

$$\boxed{y_p' = B_p \sin(\omega_v t - \delta_p)} \leftarrow \text{nastavek}$$

$$(\ddot{y}_h' + \ddot{y}_p') + \beta(\dot{y}_h' + \dot{y}_p') + \omega_0^2 (y_h' + y_p') = \frac{F_0}{m}\sin(\omega_v t)$$

$$\underbrace{\ddot{y}_h' + \beta\dot{y}_h' + \omega_0^2 y_h'}_{=0} + \ddot{y}_p' + \beta\dot{y}_p' + \omega_0^2 y_p' = \frac{F_0}{m}\sin(\omega_v t)$$

Rešujemo enačbo: $\boxed{\ddot{y}_p' + \beta\dot{y}_p' + \omega_0^2 y_p' = \frac{F_0}{m}\sin(\omega_v t)}$

$$y_p' = B_p[\sin(\omega_v t)\cos\delta - \sin\delta\cos(\omega_v t)] \leftarrow \text{NASTAVEK}$$

$$\dot{y}_p' = \omega_v B_p \cos(\omega_v t - \delta_p) = \omega_v B_p[\cos(\omega_v t)\cos\delta_p + \sin(\omega_v t)\sin\delta_p]$$

$$\ddot{y}_p' = \omega_v^2 B_p \sin(\omega_v t - \delta_p) = \omega_v^2 B_p[\sin(\omega_v t)\cos\delta_p - \sin\delta_p\cos(\omega_v t)]$$

$$\boxed{\begin{aligned} B_p\{(\omega_0^2 - \omega_v^2)[\sin(\omega_v t)\cos\delta_p - \cos(\omega_v t)\sin\delta_p] +\\ + \omega_v\beta[\cos(\omega_v t)\cos\delta_p + \sin(\omega_v t)\sin\delta_p]\} = \frac{F_0}{m}\sin(\omega_v t);\ \forall t \end{aligned}}$$

a) $t_1 = 0$: $B_p\{(\omega_v^2 - \omega_0^2)\sin\delta_p + \omega_v\beta\cos\delta_p\} = 0$

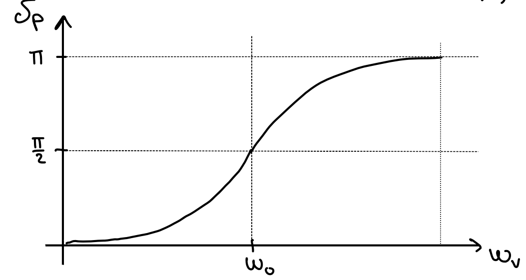
$\sin(\omega_v t_1) = 0$
$\cos(\omega_v t_1) = 1$

$$\Rightarrow \boxed{\tan\delta_p = \frac{\omega_v\beta}{\omega_0^2 - \omega_v^2}}$$

$$\Rightarrow \cos\delta_p = \pm\frac{(\omega_0^2 - \omega_v^2)}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2\beta^2}}$$

in

$$\sin\delta_p = \pm\frac{\omega_v\beta}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \omega_v^2\beta^2}}$$



$\omega_v \to 0 : \tan\delta_p \to +0 : \delta_p \to 0$
$\omega_v \nearrow \omega_0 : \tan\delta_p \to \infty : \delta_p \nearrow \frac{\pi}{2}$
$\omega_v \searrow \omega_0 : \tan\delta_p \to -\infty : \delta_p \searrow \frac{\pi}{2}$
$\omega_v \to \infty : \tan\delta_p \to -0 : \delta_p \nearrow \pi$

$\cos\delta_p : \omega_v \to 0 \Rightarrow \delta_p \to 0 : \cos\delta_p \to +1$

$\sin\delta_p : \delta_p \in [0, \pi] \Rightarrow \sin\delta_p > 0$

b) $t_2 : \omega_v t_2 = \frac{\pi}{2} \Rightarrow \sin(\omega_v t_2) = 1,\ \cos(\omega_v t_2) = 0$

$$B_p\{(\omega_0^2 - \omega_v^2)\cos\delta_p + \beta\omega_v\sin\delta_p\} = \frac{F_0}{m}$$

vstavimo sin in cos

$$B_p\left\{\frac{(\omega_0^2 - \omega_v)^2 + \beta^2\omega_v^2}{\sqrt{(\omega_0^2 - \omega_v^2)^2 + \beta^2\omega_v^2}}\right\} = \frac{F_0}{m}$$

$$\Rightarrow B_p\sqrt{(\omega_0^2 - \omega_v^2)^2 + \beta^2\omega_v^2} = \frac{F_0}{m}$$

$$\Rightarrow \boxed{B_p = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_v^2)^2 + \beta^2\omega_v^2}} = \frac{F_0}{m\omega_v}\sqrt{\frac{\omega_v^2}{\omega_0^2 - \omega_v^2 + \beta\omega_v^2}}}$$

$$|z| = \frac{F_0}{B_p\,(m\omega_v)} = \sqrt{\frac{(\omega_0^2-\omega_v^2)^2}{\omega_v^2}+\beta^2}$$ ← absolutna vrednost impendance

$$\boxed{z = |z| e^{i\delta_p}}$$ ← IMPENDANCA

Kakšen naj bo ω_v, da bo B_p maksimalne vrednosti?

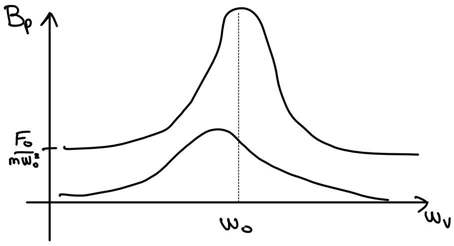
gledamo: $(\omega_0^2-\omega_v^2)^2 + \omega_v^2\beta^2 = \min.$

$$\frac{d}{d\omega_v}:\quad 2(\omega_0^2-\omega_v^2)(-2\omega_v) + 2\omega_v\beta^2 = 0$$

$$-2\omega_0^2 + 2\omega_v^2 + \beta^2 = 0$$

$$\Rightarrow \boxed{\omega_v^2 = \omega_0^2 - \frac{\beta^2}{2}}$$

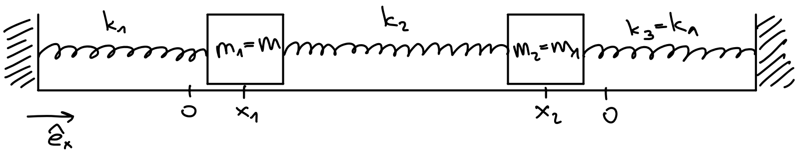
$$\Rightarrow \omega_v = \sqrt{\omega_0^2 - \frac{\beta^2}{2}} = \omega_0\sqrt{1-\frac{\beta^2}{2\omega_0^2}}$$



SKLOPLJENO NIHANJE

DVA SKLOPLJENA VOZIČKA NA ZRAČNI KLOPI

(1) simetričen primer:



$\vec{F}_{1\to 1,r} = -k_1 \Delta l_{1,r}\,\hat{e}_x$ ↖ sila 1. vzmeti na 1. voziček v ravnovesju

$\vec{F}_{2\to 1,r} = +k_2 \Delta l_{2,r}\,\hat{e}_x$

absolutno enaki

$\vec{F}_{2\to 2,r} = -k_2 \Delta l_{2,r}\,\hat{e}_x$

$\vec{F}_{3\to 2,r} = +\underset{\substack{\parallel \\ k_3}}{k_1} \Delta l_{2,r}\,\hat{e}_x$

$x_1, x_2 = 0$: $\vec{F}_{1\to 1,r} + \vec{F}_{2\to 1,r} = 0$ in $\vec{F}_{2\to 2,r} + \vec{F}_{3\to 2,r} = 0$

Oba vozička zmaknemo iz ravnovesne lege:

x_1, x_2: $\vec{F}_{1\to 1} = \vec{F}_{1\to 1,r} - k_1 x_1 \hat{e}_x$ ← enako kot v primeru enega vozička

$$\vec{F}_{2\to 1} = \vec{F}_{2\to 1,r} - k_2(x_1-x_2)\,\hat{e}_x$$

$$\boxed{\vec{F}_{1\to 1} + \vec{F}_{2\to 1} = m\ddot{x}_1\hat{e}_x}$$

$$\vec{F}_{1\to 1,r} - k_1 x_1 \hat{e}_x + \vec{F}_{2\to 1,r} - k_2 (x_1 - x_2)\hat{e}_x = m\ddot{x}_1 \hat{e}_x$$

$$\left[m\ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2)\right] \hat{e}_x = 0$$

$$\Rightarrow \boxed{\ddot{x}_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) = 0} \quad (1) \quad ; \quad \boxed{\omega_1^2 = \frac{k_1}{m} \ , \quad \omega_2^2 = \frac{k_2}{m}}$$

Podobno:

$$\Rightarrow \boxed{\ddot{x}_2 + \omega_1^2 x_2 - \omega_2^2 (x_1 - x_2) = 0} \quad (2) \qquad (k_3 = k_1, \ m_2 = m_1)$$

$$\left.\begin{aligned} x_a &\equiv x_1 + x_2 \\ x_b &\equiv x_1 - x_2 \end{aligned}\right\} \Rightarrow \begin{aligned} x_1 &= \frac{x_a + x_b}{2} \\ x_2 &= \frac{x_a + x_b}{2} \end{aligned}$$

Seštejemo (1) in (2):

$$\ddot{x}_1 + \ddot{x}_2 + \omega_1^2 (x_1 - x_2) = 0$$

$$\ddot{(x_1 + x_2)} + \omega_1^2 (x_1 - x_2) = 0$$

$$\Rightarrow \boxed{\ddot{x}_a + \omega_a^2 x_a = 0} \quad ; \quad \omega_a^2 = \omega_1^2$$

nesklopljeni enačbi

Odštejemo (1) in (2):

$$\Rightarrow \boxed{\ddot{x}_b + \omega_b^2 x_b = 0} \quad ; \quad \omega_b^2 = \omega_1^2 + 2\omega_2^2$$

$$\boxed{\begin{aligned} x_a &= B_a \sin(\omega_a t + \delta_a) \\ x_b &= B_b \sin(\omega_b t + \delta_b) \end{aligned}}$$

$$\Rightarrow \boxed{\begin{aligned} x_1 &= C_1 \sin(\omega_a t + \delta_a) + C_2 \sin(\omega_b t + \delta_b) \\ x_2 &= C_1 \sin(\omega_a t + \delta_a) - C_2 \sin(\omega_b t + \delta_b) \end{aligned}} \quad ; \quad C_1 = \frac{B_a}{2} \ , \quad C_2 = \frac{B_b}{2}$$

$C_1, C_2, \delta_a, \delta_b$ iz začetnih pogojev: $x_1(t=0)$, $\dot{x}_1(t=0)$, $x_2(t=0)$, $\dot{x}_2(t=0)$

Primer:

$$\begin{aligned} x_1(t=0) &= x_0 \ ; & \dot{x}_1(t=0) &= 0 \\ x_2(t=0) &= 0 & \dot{x}_2(t=0) &= 0 \end{aligned}$$

$$\boxed{\begin{aligned} \dot{x}_1 &= C_1 \omega_a \cos(\omega_a t + \delta_a) + C_2 \omega_b \cos(\omega_b t + \delta_b) \\ \dot{x}_2 &= C_1 \omega_a \cos(\omega_a t + \delta_a) - C_2 \omega_b \cos(\omega_b t + \delta_b) \end{aligned}}$$

$$\begin{aligned} x_0 &= C_1 \sin\delta_a + C_2 \sin\delta_b & \qquad 0 &= C_1 \omega_a \cos\delta_a + C_2 \omega_b \cos\delta_b \\ 0 &= C_1 \sin\delta_a - C_2 \sin\delta_b & 0 &= C_1 \omega_a \cos\delta_a - C_2 \omega_b \cos\delta_b \end{aligned}$$

$$\left.\begin{aligned} 2C_1 \omega_a \cos\delta_a &= 0 \\ 2C_2 \omega_b \cos\delta_b &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} \cos\delta_a &= 0 \Rightarrow \delta_a = \pm\frac{\pi}{2} \\ \cos\delta_b &= 0 \Rightarrow \delta_b = \pm\frac{\pi}{2} \end{aligned}$$

(1) $\delta_a = \delta_b = \frac{\pi}{2}$:

$$\Rightarrow \sin\delta_a = 1 = \sin\delta_b$$

$$\left.\begin{aligned} x_0 &= C_1 + C_2 \\ 0 &= C_1 - C_2 \end{aligned}\right\} \Rightarrow C_1 = C_2 = \frac{x_0}{2}$$

$$\Rightarrow \boxed{\begin{aligned} x_1 &= \frac{x_0}{2}\left[\cos(\omega_a t) + \cos(\omega_b t)\right] = x_0 \cos\left(\frac{\omega_a + \omega_b}{2} t\right) \cos\left(\frac{\omega_a - \omega_b}{2} t\right) \\ x_2 &= \frac{x_0}{2}\left[\cos(\omega_a t) - \cos(\omega_b t)\right] = x_0 \sin\left(\frac{\omega_a + \omega_b}{2} t\right) \sin\left(\frac{\omega_a - \omega_b}{2} t\right) \end{aligned}}$$

Šibka sklopitev: $k_2 \ll k_1 \Rightarrow \omega_2 = \frac{k_2}{m} \ll \omega_1 = \frac{k_1}{m}$

$$\omega_b = \sqrt{\omega_1^2 + 2\omega_2^2} = \omega_1 \sqrt{1 + 2\frac{\omega_2^2}{\omega_1^2}} \sim \omega_1 \sqrt{1 + 2\left(\frac{\omega_2}{\omega_1}\right)^2} = \omega_1 + \omega_1 \frac{\omega_2^2}{\omega_1^2}$$

$$\Rightarrow \frac{\omega_a + \omega_b}{2} \simeq \omega_1$$

$$\frac{\omega_a - \omega_b}{2} \simeq \omega_1 \frac{\omega_2^2}{\omega_1^2} \ll \omega_1$$

$$\Rightarrow \boxed{x_o \doteq x_o \cos(\omega_1 t) \cos\left(\omega_1 \frac{\omega_2^2}{\omega_1^2} t\right)}$$

LASTNA NIHANJA SESTAVLJENEGA NIHALA

$$\left.\begin{array}{l} x_1 = B_1 \sin(\omega_a t + \delta_a) + B_2 \sin(\omega_b t + \delta_b) \\ x_2 = B_1 \sin(\omega_a t + \delta_a) - B_2 \sin(\omega_b t + \delta_b) \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} x_1 = B_1 \sin(\omega_L t + \delta_1) \\ x_2 = B_2 \sin(\omega_L t + \delta_2) \end{array}} \; ; \; \nu_L = \frac{\omega_L}{2\pi}$$

Osnovni enačbi:

$$\boxed{\begin{array}{l} \ddot{x}_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) = 0 \\ \ddot{x}_2 + \omega_1^2 x_2 - \omega_2^2 (x_1 - x_2) = 0 \end{array}} \begin{array}{l} ; \; \omega_1^2 = \frac{k_1}{m} \\ ; \; \omega_2^2 = \frac{k_2}{m} \end{array}$$

Nastavek: $\left.\begin{array}{l} x_1 = C e^{i\lambda t} \\ x_2 = D e^{i\lambda t} \end{array}\right\} \Rightarrow \begin{array}{l} \ddot{x}_1 = -\lambda^2 x_1 \\ \ddot{x}_2 = -\lambda^2 x_2 \end{array}$

$$\left.\begin{array}{l} \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} C \\ D \end{bmatrix} e^{i\lambda t} \\ -\lambda^2 x_1 + \omega_1^2 x_1 + \omega_2^2 (x_1 - x_2) = 0 \\ -\lambda^2 x_2 + \omega_1^2 x_2 + \omega_2^2 (x_1 - x_2) = 0 \end{array}\right\} \Rightarrow \begin{bmatrix} \omega_1^2 + \omega_2^2 - \lambda^2 & -\omega_2^2 \\ -\omega_2^2 & \omega_1^2 + \omega_2^2 - \lambda^2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} e^{i\lambda t} = 0 \; ; \; \forall t$$

$$\begin{bmatrix} \omega_1^2 + \omega_2^2 - \lambda^2 & -\omega_2^2 \\ -\omega_2^2 & \omega_1^2 + \omega_2^2 - \lambda^2 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = 0 \qquad (C, D, \lambda = ?)$$

$$\Rightarrow (\omega_1^2 + \omega_2^2 - \lambda^2)^2 - \omega_2^2 \omega_2^2 = 0$$

$$\omega_1^2 + \omega_2^2 - \lambda^2 = \pm \omega_2^2$$

a) $+$: $\lambda^2 = \omega_1^2 \Rightarrow \boxed{\lambda = \pm \omega_1 = \pm \omega_a}$

b) $-$: $\underbrace{\omega_1^2 + 2\omega_2^2}_{\omega_b^2} = \lambda^2 \Rightarrow \boxed{\lambda = \pm \omega_b}$

a) $\lambda = \pm \omega_a$: $\quad +)$: $\begin{array}{l} x_1 = C_1 e^{i\omega_1 t} \\ x_2 = D_1 e^{i\omega_1 t} \end{array}$ $\quad \begin{bmatrix} \omega_2^2 & -\omega_2^2 \\ -\omega_2^2 & \omega_2^2 \end{bmatrix} \begin{bmatrix} C_1 \\ D_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\left.\begin{array}{l} \omega_2^2 (C_1 - D_1) = 0 \\ -\omega_2^2 (C_1 - D_1) = 0 \end{array}\right\} \Rightarrow \boxed{C_1 = D_1}$$

dobimo na enak način
↓

$-)$: $\quad C_2 = D_2$: $\quad \begin{array}{l} x_1 = C_2 e^{-i\omega_1 t} \\ x_2 = D_2 e^{-i\omega_1 t} \end{array}$

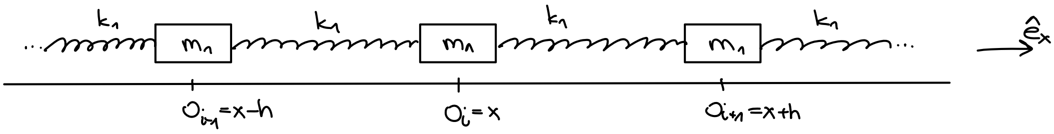
$$\Rightarrow \boxed{\begin{array}{l} x_1 = C_1 e^{i\omega_1 t} + C_2 e^{-i\omega_1 t} \\ x_2 = C_1 e^{i\omega_1 t} + C_2 e^{-i\omega_1 t} \end{array}}$$

$$\Rightarrow \begin{array}{l} x_1 = (C_1 + C_2)\cos(\omega_1 t) + i(C_1 - C_2)\sin(\omega_1 t) = B_1 \sin(\omega_L t + \delta_1) \quad \forall t \\ x_2 = x_1 = B_2 \sin(\omega_L t + \delta_2) \quad \forall t \end{array}$$

$$\left.\begin{array}{l} \text{(i)}\ t_1=0: \quad C_1+C_2 = B_1 \sin\delta_1 \\ \qquad\qquad\quad C_1+C_2 = B_2 \sin\delta_2 \\ \\ \quad B_1 \sin(\omega_2 t+\delta_1) = B_2 \sin(\omega_2 t+\delta_2) \end{array}\right\} \Rightarrow B_1 = B_2 \ \text{ in } \ \delta_1=\delta_2$$

MEHANSKO VALOVANJE - VALOVNA ENAČBA

DISKRETNI MODEL MASIVNE VZMETI:



ℓ ... skupna dolžina vzmeti

$nm_1=m$ ... masa vzmeti

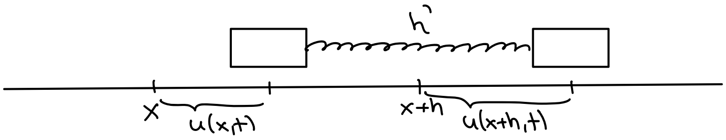
k ... koeficient celotne vzmeti

$$n=\frac{\ell}{h}, \quad m_1=\frac{m}{n}, \quad k_1=k\frac{\ell}{h}$$

$x \mapsto u(x,t)$ ... odmik uteži z ravnovesno lego v x od ravnovesne lege ob času t v smeri $\hat{e}_x$

$x \neq x(t)$ oz. $\dot{x}=0$

ravnovesna lega se s časom ne spreminja



$$u(x,t)+h' = h+u(x+h,t)$$

$$\Rightarrow h'-h = \boxed{\Delta h = u(x+h,t)-u(x,t)}$$

$$\left.\begin{array}{l} \Rightarrow F_{i+1\to i} = F_{i+1\to i,r} + k_1[u(x+h,t)-u(x,t)] \\ \quad \text{sila } i+1 \text{ uteži na } i\text{-to utež preko vzmeti} \\ F_{i-1\to i} = F_{i-1\to i,r} - k_1[u(x,t)-u(x-h,t)] \end{array}\right\} \Rightarrow F_{i+1\to i,r} + F_{i-1\to i,r} = 0$$

$$\Rightarrow F_i = F_{i+1\to i} + F_{i-1\to i} = k_1[\underbrace{u(x+h,t)-u(x,t)}_{h} - \underbrace{u(x,t)+u(x-h,t)}_{h}] =$$

skupna sila na telo i

$$= k\ell\left[\frac{u(x+h,t)-u(x,t)}{h} - \frac{u(x,t)-u(x-h,t)}{h}\right]$$

$$F_i = m_1\frac{d^2u(x,t)}{dt^2} \quad \text{(2. Newtonov zakon)}$$

$$\frac{du(x,t)}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\underbrace{\frac{\partial x}{\partial t}}_{\dot{x}=0} = \frac{\partial u}{\partial t}$$

$$\Rightarrow \boxed{\frac{d^2u}{dt^2} = \frac{d}{dt}\left(\frac{\partial u}{\partial t}\right) = \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x \partial t}\dot{x} = \frac{\partial^2 u}{\partial t^2}}$$

$$\Rightarrow \boxed{F_i = m_1\frac{\partial^2 u}{\partial t^2}}\ ; \quad m_1 = \frac{m}{n} = \frac{m}{\ell}h$$

$$\frac{m}{\ell}\frac{\partial^2 u}{\partial t^2} = k\ell\,\frac{\frac{u(x+h,t)-u(x,t)}{h} - \frac{u(x,t)-u(x-h,t)}{h}}{h}$$

ni nujno da obstaja limita tega

$$\Rightarrow \frac{\partial^2 u}{\partial t^2} = \frac{k\ell^2}{m}\,\frac{\frac{u(x+h,t)-u(x,t)}{h} - \frac{u(x,t)-u(x-h,t)}{h}}{h}$$

$$n \to \infty \Leftrightarrow h \to 0: \quad \frac{\frac{\partial u(x,t)}{\partial x} - \frac{\partial u(x-h,t)}{\partial x}}{h} \Rightarrow \frac{\partial^2 u(x,t)}{\partial x^2}$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}} \leftarrow \text{VALOVNA ENAČBA}$$

$$c^2 = \frac{k\ell^2}{m} \quad \left[\frac{m}{s}\right]$$

VALOVANJE PO ELASTIČNI PALICI:

$$\frac{\Delta F}{S} = -E \frac{\Delta \ell}{\ell}; \qquad [E] = \frac{N}{m^2}$$

JOUNGOV ELASTIČNI MODUL

$$\Delta \ell \propto \frac{1}{E}$$

$$\Delta F = -\underbrace{\frac{ES}{\ell}}_{k} \Delta \ell = k \Delta \ell \qquad \boxed{c^2 = \frac{k\ell^2}{m} = \frac{ES\ell^2}{\ell m} = \frac{ES\ell}{m} = \frac{E \cdot V}{m} = \frac{E}{\rho}}$$

$$\Delta F > 0. \quad \Delta \ell < 0$$

TEKOČINA, ZAPRTA V DOLGI TANKI CEVI:

$\eta \to 0$ ← viskoznost

$S = \text{konst.}$

$$\Delta p = -\frac{1}{\chi}\frac{\Delta V}{V}; \quad \Delta V = -\chi V \Delta p$$

$$\Delta p > 0 \Rightarrow \Delta V < 0$$

$$\Delta p < 0 \Rightarrow \Delta V > 0$$

$$\frac{\Delta F}{S} = -\frac{1}{\chi}\frac{\Delta V}{V}$$

$$\Delta F = \frac{-S}{\chi V}\Delta V; \quad \Delta V = S \Delta \ell$$

$$\Rightarrow \Delta F = -\frac{S S \Delta \ell}{\chi S \ell}$$

$$\Delta F = -\underbrace{\frac{S}{\chi \ell}}_{k} \Delta \ell = k \cdot \Delta \ell$$

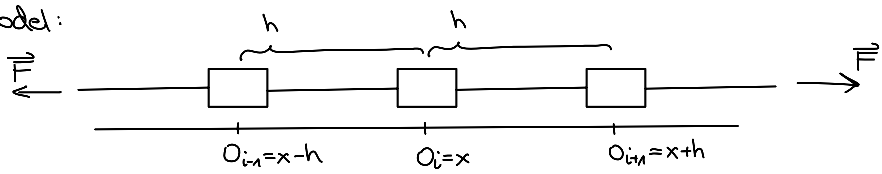
$$\boxed{c^2 = \frac{k\ell^2}{m} = \frac{S\ell^2}{\chi \ell m} = \frac{S\ell}{\chi m} = \frac{V}{\chi m} = \frac{1}{\chi S}}$$

Do sedaj: $u(x,t)$ v smeri $\hat{e}_x$ (smer potovanja motnje) $\Rightarrow$ $\boxed{\text{LONGITUDINALNO VALOVANJE}}$

VALOVANJE PO NAPETI VRVI:

$u(x,t) \perp \hat{e}_x$

Diskretni model:



$u(x,t)$: odmik $(i-k)$-te uteži od ravnovesne lege ob času t v smeri $\hat{e}_y$

$$\Rightarrow \boxed{\frac{\partial^2 u(x,t)}{\partial t^2} = c^2 \frac{\partial^2 u(x,t)}{\partial x^2}; \quad c^2 = \frac{F}{\frac{m}{\ell}}}$$

VALOVNA ENAČBA V 3 DIMENZIJAH:

$$\frac{\partial^2}{\partial x^2} \longrightarrow \nabla^2 \equiv \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u}$$

Definiramo: $\square \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$

$$\square u = 0$$

REŠITVE 1-DIMENZIONALNE VALOVNE ENAČBE

dvakrat odvedljiva $f(x-ct)$

$u(x,t) = f(x-ct)$; $\eta \equiv x - ct$

$\Rightarrow u = f(\eta(x,t))$

$$\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial \eta}\right) = \frac{\partial}{\partial \eta}\left(\frac{\partial u}{\partial \eta}\right)\frac{\partial \eta}{\partial t}$$

$$\Rightarrow \frac{\partial u}{\partial t} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial t} = -c\frac{\partial u}{\partial \eta}$$

$$\Rightarrow \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}\left(-c\frac{\partial u}{\partial \eta}\right) = -c\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial \eta}\right) = -c\frac{\partial}{\partial \eta}\left(\frac{\partial u}{\partial \eta}\right)\frac{\partial \eta}{\partial t} = c^2\frac{\partial^2 u}{\partial \eta^2}$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial \eta^2}$$

$$\Rightarrow \frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$$

INTERPRETACIJA REŠITVE:

Velja:

a) $u(x,t) = u(x-ct, 0)$

b) $u(x,t) = u(0, t - \frac{x}{c})$

* velja tudi: $u(x,t) = g(x+ct)$

$\Rightarrow u(x,t) = u_1(x,t) + u_2(x,t)$ tudi rešitev

$u(x,t) = f(x-ct)$:

a) $u(x-ct, 0) = f(x - ct - c\cdot 0) = f(x-ct) = u(x,t)$

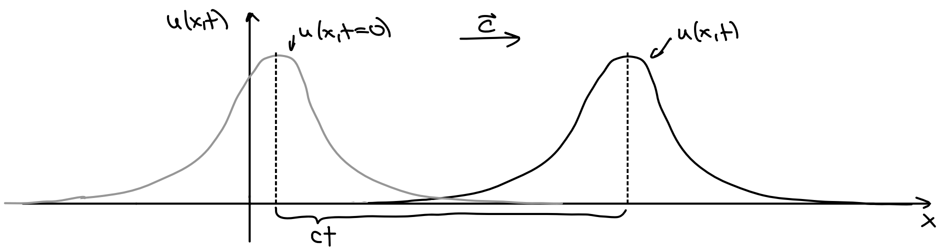
b) $u(0, t-\frac{x}{c}) = f(0 - c(t - \frac{x}{c})) = f(-ct+x) = f(x-ct) = u(x,t)$

Iz a):

$u(x,t)$ ... odmik od ravnovesne lege ob času t

$\shortparallel$

$u(x-ct, 0)$ ... odmik točke iz ravnovesne lege $x-ct$ ob času 0



t: motnja prepotuje razdaljo $ct = \Delta x$

$\Rightarrow c$ ... hitrost potovanja motnje / valovanja

c: v splošnem ni enaka hitrosti v $\left(v = \left|\frac{du}{dt}\right| \overset{\dot{x}=0}{=} \left|\frac{\partial u}{\partial t}\right| = \left|\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial t}\right| = c\left|\underset{\neq 1}{\frac{\partial u}{\partial \eta}}\right| \Rightarrow v \neq c\right)$

(a) $\Rightarrow$ val točno določen iz začetnega pogoja

(b): če poznamo $u(x=0,t)$, lahko določimo $u(x,t)$
$\Rightarrow$ val določen iz robnega pogoja

** $\boxed{u(x,t) = u_1(x,t) + u_2(x,t) \quad \text{tudi rešitev}}$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial^2}{\partial t^2}(u_1+u_2) = \frac{\partial^2 u_1}{\partial t^2} + \frac{\partial^2 u_2}{\partial t^2} = c^2 \frac{\partial^2 u_1}{\partial x^2} + c^2 \frac{\partial^2 u_2}{\partial x^2}$$

$$\frac{\partial^2}{\partial x^2}(u_1+u_2) = \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_2}{\partial x^2}$$

$$\boxed{u(x,t) = f(x-ct) + g(x+ct)}$$

Dva začetna pogoja (d'Alembert)

$$u(x,0) = f(x) + g(x) = A(x)$$

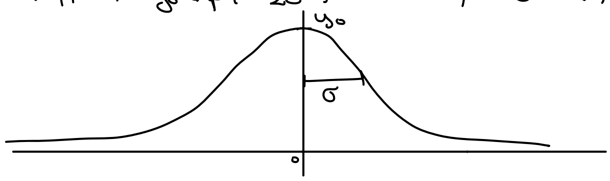
$$\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = B(x)$$

$\vdots$

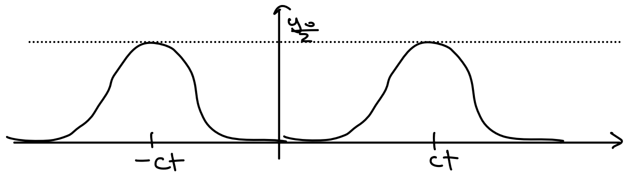
$$u(x,t) = \frac{1}{2}\left[A(x-ct) + A(x+ct)\right] + \frac{1}{2c}\int_{x-ct}^{x+ct} B(x)\,dx$$

Primer:

$$u(x,t=0) = y_0 \exp\left\{-\frac{x^2}{2\sigma^2}\right\} = A(x) \quad ; \quad 0 = B(x)$$



$$u(x,t) = \frac{y_0}{2}\left[\exp\left\{-\frac{(x-ct)^2}{2\sigma^2}\right\} + \exp\left\{\frac{-(x+ct)^2}{2\sigma^2}\right\}\right]$$



POTUJOČE SINUSNO VALOVANJE

$$u(x,t) = f(x-ct)$$

a) $u(x,t) = u(x-ct, 0)$ (začetni pogoji)

b) $u(x,t) = u(0, t-\frac{x}{c})$ (robni pogoji)

$u(x=0, t) = u_0 \sin(-\omega t + \delta)$; $\forall t$

$$\boxed{u(x,t) = u(0, t-\tfrac{x}{c}) = u_0 \sin\left(-\omega(t-\tfrac{x}{c}) + \delta\right) = u_0 \sin\left(\underbrace{\tfrac{\omega}{c}}_{k} x - \omega t + \delta\right) = u_0 \sin(kx - \omega t + \delta)}$$

$k = \frac{\omega}{c} = \frac{2\pi\nu}{c}$ $\left[\frac{1}{m}\right]$

VALOVNO ŠTEVILO

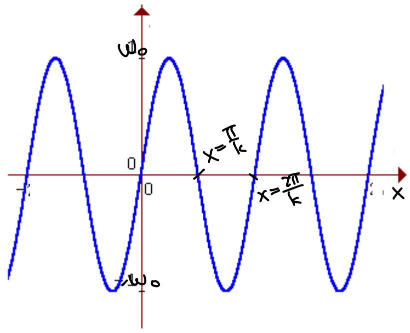
Primer: $\delta = 0$: slikamo ob času $t=0$

$u(x, t=0) = u_0 \sin(kx)$

$\lambda = \frac{2\pi}{k}$

$\Rightarrow k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c}$

$\Rightarrow \boxed{c = \lambda\nu}$



(valovanje v drugo smer: $u(x,t) = g(x+ct)$)

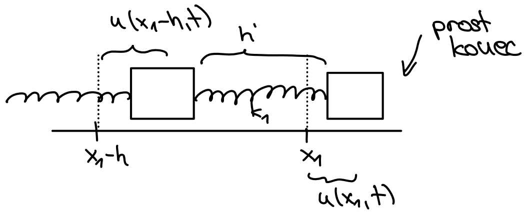
ROBNI POGOJI IN STOJNO/STOJEČE SINUSNO VALOVANJE

ROBNI POGOJI:

a) $u(x_1, t) = 0$ (togo vpet konec)

b) $\frac{\partial u}{\partial x}(x,t) = 0$ (prost konec)

MODEL DISKRETNE MASIVNE VZMETI:



$u(x_1-h,t) + h' = h + u(x_1,t)$

$\Delta h = h' - h = u(x_1,t) - u(x_1-h,t)$

$F_{x_1-h \to x_1} = \underbrace{F_{x_1-h \to x_1, r}}_{0} - k_1 \Delta h = -k_1 \Delta h = \boxed{-\frac{k\ell}{h}\left(u(x_1,t) - u(x_1-h,t)\right) = F_{x_1}}$

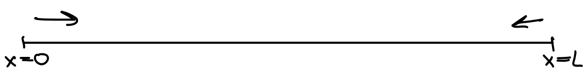
2. Newtonov zakon: $\boxed{F_{x_1} = m_1 \frac{\partial^2 u(x,t)}{\partial t^2} = \frac{m_1 \partial^2 u}{\partial t^2} = \frac{m}{n}\frac{\partial^2 u}{\partial t^2} = \frac{mh}{\ell}\frac{\partial^2 u}{\partial t^2}}$

$\Rightarrow -\underbrace{\frac{k\ell^2}{m}}_{c^2} \frac{u(x_1,t) - u(x_1-h,t)}{h} = h\frac{\partial^2 u}{\partial t^2}$

$n \to \infty, h \to 0$: $-c^2 \frac{\partial u(x,t)}{\partial x}\Big|_{x=x_1} = 0 \Rightarrow \boxed{\frac{\partial u(x,t)}{\partial x}\Big|_{x=x_1} = 0}$

PALICA PROSTA NA OBEH KONCIH:

$\frac{\partial u}{\partial x}(x,t)_{x=0} = 0$ in $\frac{\partial u}{\partial x}(x,t)_{x=L} = 0$; L ... dolžina palice



Predpostavimo: $u(x,t) = u_0 \sin(kx - \omega t + \delta_1) + u_0 \sin(kx + \omega t + \delta_2)$

$\frac{\partial u}{\partial x}(x,t) = u_0 k[\cos(kx - \omega t + \delta_1) + \cos(kx + \omega t + \delta_2)] =$

$= 2u_0 k \cos\left(kx + \frac{\delta_1+\delta_2}{2}\right)\cos\left(-\omega t + \frac{\delta_2 - \delta_1}{2}\right)$

a) $x=0$: $2u_0 k \cos\left(\frac{\delta_1+\delta_2}{2}\right)\cos\left(-\omega t + \frac{\delta_1-\delta_2}{2}\right) = 0, \ \forall t \Rightarrow \frac{\delta_1+\delta_2}{2} = \frac{\pi}{2}$

$\cos\left(kx + \frac{\delta_1+\delta_2}{2}\right) = \cos\left(kx + \frac{\pi}{2}\right) = -\sin(kx)$

$$x=L: \quad 2u_0 k \sin(kL)\cos\left(-\omega t+\frac{\delta_1-\delta_2}{2}\right)=0,\ \forall t \Rightarrow kL=n\pi,\ n\in\mathbb{Z}$$
$$\Rightarrow k_n=\frac{n\pi}{L}$$

$$k_n=\frac{2\pi}{\lambda_n}=\frac{n\pi}{L} \rightarrow \frac{2}{\lambda_n}=\frac{n}{L}$$
$$\Rightarrow \boxed{\lambda_n=\frac{2L}{n}}\boxed{=\frac{1}{\nu_n}}$$

$$\lambda_n\nu_n=c \Rightarrow \boxed{\nu_n=\frac{nc}{2L}}$$

$$u(x,t)=2u_0\sin\left(kx+\frac{\delta_1+\delta_2}{2}\right)\cos\left(-\omega t+\frac{\delta_1-\delta_2}{2}\right)=$$
$$=2u_0\sin\left(kx+\frac{\pi}{2}\right)\cos\left(\omega t+\frac{\delta_2-\delta_1}{2}\right)=$$
$$=2u_0\cos(k_nx)\cos\left(\omega_n t+\frac{\delta_2-\delta_1}{2}\right) \quad ; \quad k_n=\frac{\omega_n}{c}$$

Primer: $\frac{\delta_2-\delta_1}{2}=0$

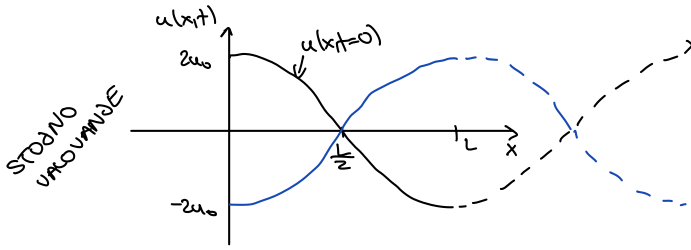
$$\Rightarrow u(x,t)=2u_0\cos(k_nx)\cos(\omega t)$$

$t=0$:

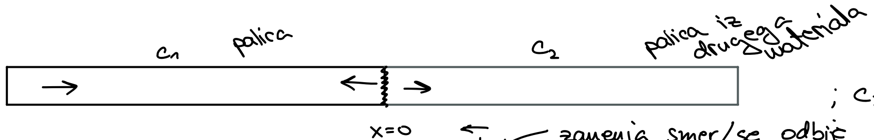
$n=1$: $k_1=\frac{\pi}{L} \Rightarrow \cos(k_nx)=\cos\left(\frac{\pi x}{L}\right)$; $x=0\Rightarrow \cos\left(\frac{\pi}{L}\cdot 0\right)=1$

$x=L\Rightarrow \cos\left(\frac{\pi}{L}\cdot L\right)=-1$

$x=\frac{L}{2}\Rightarrow \cos\left(\frac{\pi}{L}\cdot\frac{L}{2}\right)=0$



Primer:



; $c_2\neq c_1$

zamenja smer/se odbije ← na eni strani

$x\le 0$: $u_1(x,t)=f_1(x-c_1t)+g_1(x+c_1t)$

$x\ge 0$: $u_2(x,t)=f_2(x-c_2t)$

odbojnost: $\frac{|g_1|}{|f_1|}=\frac{|c_1-c_2|}{c_1+c_2}$; $c_2=0 \Rightarrow \frac{|g_1|}{|f_1|}=1$

prepustnost: $\frac{|f_2|}{|f_1|}=\frac{2c_2}{c_1+c_2}$ $\quad c_2=0 \Rightarrow \frac{|f_2|}{|f_1|}=0$

} vse se odbije

$$\boxed{\begin{aligned}&u_1(x=0,t)=u_2(x=0,t)\\&\frac{\partial u_1}{\partial x}(x=0,t)=\frac{\partial u_2}{\partial x}(x=0,t)\end{aligned}}$$ (palica se ne strga)

ENERGIJA VALOVANJA

Potovanje motnje → potovanje energije

diskretni model vzmeti: W_k ; W_{pr}

utež na položaju x

x: $W_k=\frac{1}{2}m_1v^2$
$$=\frac{1}{2}m_1\left(\frac{du(x,t)}{dt}\right)^2$$
$$=\frac{1}{2}m_1\left(\frac{\partial u(x,t)}{\partial t}\right)^2$$
$$=\frac{1}{2}\frac{mh}{\ell}\left(\frac{\partial u(x,t)}{\partial t}\right)^2$$

$\Rightarrow \boxed{\frac{W_k}{h}=\frac{1}{2}\frac{m}{\ell}\left(\frac{\partial u(x,t)}{\partial t}\right)^2}$ ← kinetična energija posamezne uteži

; h… razdalja med sosednjima utežema v ravnovesju

$$W_{pr}=\frac{1}{2}k_1(\Delta h)^2=\frac{1}{2}k_1\left[u(x+h,t)-u(x,t)\right]^2=\frac{1}{2}\frac{k\ell}{h}\left[u(x+h,t)-u(x,t)\right]^2=\frac{1}{2}k\ell h\left[\frac{u(x+h,t)-u(x,t)}{h}\right]^2$$

$$\Rightarrow \frac{W_{pr}}{h}=\frac{1}{2}k\ell\left[\frac{u(x+h,t)-u(x,t)}{h}\right]^2 \xrightarrow{h\to 0} \boxed{\lim_{h\to 0}\frac{W_{pr}}{h}=\frac{1}{2}k\ell\left[\frac{\partial u(x,t)}{\partial x}\right]^2}$$ ← prožnostna energija

$$\frac{1}{2}kl = \frac{1}{2}kl\,\frac{l}{m}\,\frac{m}{l} = \frac{1}{2}\,\frac{kl^2}{m}\,\frac{m}{l} = \frac{1}{2}c^2\frac{m}{l}$$

$$\Rightarrow \lim_{h\to 0}\frac{W_{pr}}{h} = \frac{1}{2}c^2\frac{m}{l}\left(\frac{\partial u(x,t)}{\partial x}\right)^2$$

Primer prožne palice: $m=\rho V=\rho S l$
$k=\frac{ES}{l}$

$$\Rightarrow \boxed{\frac{W_k}{h} = \frac{1}{2}\rho S\left(\frac{\partial u(x,t)}{\partial t}\right)^2}$$

$$\Rightarrow \boxed{\lim_{h\to 0}\frac{W_{pr}}{h} = \frac{1}{2}c^2\rho S\left(\frac{\partial u(x,t)}{\partial x}\right)^2}$$

GOSTOTA KINETIČNE ENERGIJE: $\boxed{w_k = \frac{W_k}{\Delta V} = \frac{W_k}{hS} = \frac{1}{2}\rho\left(\frac{\partial u(x,t)}{\partial t}\right)^2}$

GOSTOTA PROŽNOSTNE ENERGIJE: $\boxed{w_{pr} = \frac{W_{pr}}{\Delta V} = \frac{W_{pr}}{Sh} = \frac{1}{2}c^2\rho\left(\frac{\partial u(x,t)}{\partial x}\right)^2}$

ENERGIJA SINUSNEGA VALOVANJA:

$$u(x,t) = u_0 \sin(kx - \omega t + \delta)$$

smer: $\hat{e}_x$

k... VALOVNO ŠTEVILO (ne koeficient vzmeti)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{\lambda\nu} = \frac{\omega}{c} \rightarrow \omega = ck$$

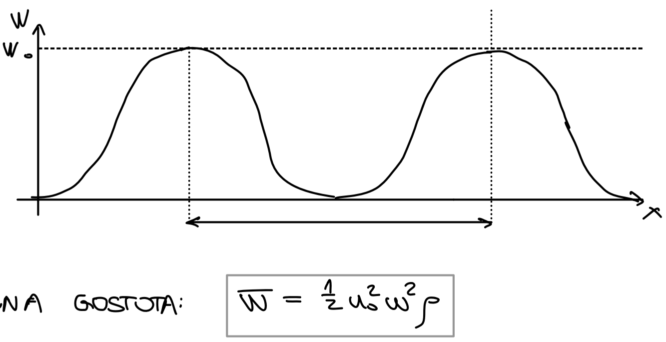
$$\boxed{\begin{aligned}\frac{\partial u(x,t)}{\partial t} &= -\omega u_0\cos(kx-\omega t+\delta)\\ \frac{\partial u(x,t)}{\partial x} &= k u_0\cos(kx-\omega t+\delta)\end{aligned}}$$

$$\Rightarrow w_k = \frac{1}{2}\rho\omega^2 u_0^2\cos^2(kx-\omega t+\delta)$$

$$w_{pr} = \frac{1}{2}c^2\rho k^2 u_0^2\cos^2(kx-\omega t+\delta) = \frac{1}{2}\omega^2\rho u_0^2\cos^2(kx-\omega t+\delta)$$

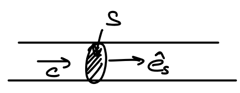
$$\Rightarrow w_{pr} = w_k$$

SKUPNA GOSTOTA: $\boxed{w = w_k + w_{pr} = u_0^2\omega^2\rho\cos^2(kx-\omega t+\delta)}$



POVPREČNA GOSTOTA: $\boxed{\overline{w} = \frac{1}{2}u_0^2\omega^2\rho}$

$\overline{W} = \Delta V\,\overline{w} = ctS\overline{w}$; $S \perp c$



$S \rightarrow \vec{S} = S\hat{e}_s$

$\hat{e}_s \parallel \hat{e}_c$

P... ENERGIJSKI TOK VALOVANJA

$$\boxed{P = \frac{\overline{W}}{t} = \frac{ctS\overline{w}}{t} = cS\overline{w}} \quad \left[W = \frac{m}{s}m^2\,\frac{m^2\,kg}{s^2\,m^3} = \frac{m^2\,kg}{s^3} = N\frac{m}{s} = \frac{J}{s}\right]$$

GOSTOTA ENERGIJSKEGA TOKA: $\boxed{j = \frac{P}{S} = \overline{w}c}$ $\left[\frac{W}{m^2}\right]$

Primeri: $j = 10^{-12}\ \frac{W}{m^2}$; $\nu = 1\ kHz$ (meja slišnosti)

$j = 1\ \frac{W}{m^2}$; $\nu = 1\ kHz$ (meja bolečine)

$j = 1{,}4\ \frac{kW}{m^2}$ (sončni tok pred vstopom v atmosfero)

Zanima nas povezava med $u(x,t)$ in Δp (tlak)

2. Newtonov zakon: $m_1 \frac{d^2u}{dt^2} = F_x$ (utež v ravnovesju v legi x)

$$\frac{mh}{l}\frac{\partial^2 u}{\partial t^2} = F_x \ ; \ F_x = F_{x-h\to x} + F_{x+h\to x}$$

$h \to 0$: $\boxed{F_x \to 0}$

$$F_{x-h\to x} + F_{x+h\to x} = 0 \ \longrightarrow \ |F_{x-h\to x}| = |F_{x+h\to x}|$$

$$F_{x+h\to x} = p \cdot S$$

$$\Rightarrow p = \frac{1}{S} F_{x-h\to x} = \frac{1}{S}\left[F_{x-h\to x, r} - k_1\left(u(x,t) - u(x-h,t)\right)\right] =$$

$$= \underbrace{\frac{F_{x-h\to x,r}}{S}}_{\substack{\text{RAVNOVESNI TLAK} \\ p_0}} - \frac{k_1\left[u(x,t)-u(x-h,t)\right]}{S}$$

$$\Rightarrow \underset{\uparrow}{p - p_0} = -\frac{k_1}{S}\left[u(x,t) - u(x-h,t)\right] = -\frac{kl}{Sh}\left[u(x,t) - u(x-h,t)\right] = -\underbrace{\frac{kl}{S}}_{\frac{1}{\chi}}\left(\frac{u(x,t)-u(x-h,t)}{h}\right) = (*)$$

razlika med trenutnim in ravnovesnim tlakom

$$\frac{1}{\chi}: \ c^2 = \frac{E}{\rho_1} = \frac{1}{\chi\rho} = \frac{kl^2}{m} \ \Rightarrow \ \frac{1}{\chi\rho} = \frac{kl^2}{m} = \frac{kl^2}{\rho S l} = \frac{kl}{\rho S}$$

$$= \frac{1}{\chi} = \frac{kl}{S}$$

$h \to 0$

$$(*) = \boxed{-\frac{1}{\chi}\frac{\partial u(x,t)}{\partial x} = p - p_0}$$

$$u = u_0 \sin(kx - \omega t + \delta)$$

$$\frac{\partial u}{\partial x} = k u_0 \cos(kx - \omega t + \delta)$$

$$\Delta p = -\frac{k}{\chi} u_0 \cos(kx - \omega t + \delta) =$$

$$= -\underbrace{\frac{\omega}{\chi c} u_0}_{\Delta p_0} \cos(kx - \omega t + \delta)$$

amplituda tlačne razlike

$$\boxed{\begin{aligned} &\Delta p = \frac{\omega u_0}{\chi c} \ ; \ \omega = 2\pi\nu \\ &j = \overline{w} \cdot c = \frac{1}{2}\rho\omega^2 u_0^2 \end{aligned}}$$

STISLJIVOST: $\boxed{\chi = -\frac{1}{V}\frac{\partial V}{\partial p}}$; $\chi > 0$

a) IZOTERMNO STISKANJE IDEALNEGA PLINA

$$\text{konst.} = \frac{pV}{T} = \frac{p'V'}{T} \ ; \quad p, V, T \longrightarrow p', V', T'$$

IZOTERMNA SPREMEMBA:

$T = T' \Rightarrow pV = p'V' = \text{konst.} \rightarrow V = \frac{K}{p}$ (K ← konstanta) $\rightarrow \frac{\partial V}{\partial p} = -\frac{K}{p^2} = -\frac{1}{p}\left(\frac{K}{p}\right) = \frac{-V}{p}$

$\Rightarrow \boxed{\chi = \frac{1}{p}}$

b) ADIABATNA / IZENTROPNA STISLJIVOST

$pV^{\kappa} = K$ (K ← konstanta) ; $\kappa = \frac{c_p}{c_v} = \begin{cases} \frac{5}{3} \doteq 1'67 ; & \text{1-atomni plin} \\ \frac{7}{5} \doteq 1'4 ; & \text{2-atomni plin} \\ \frac{4}{3} \doteq 1'3 ; & \text{3-atomni plin} \end{cases}$

$\Rightarrow V = K^{\frac{1}{\kappa}} p^{-\frac{1}{\kappa}} \Rightarrow \frac{\partial V}{\partial p} = K^{\frac{1}{\kappa}}\left(-\frac{1}{\kappa}\right)\frac{1}{p}\, p^{-\frac{1}{\kappa}} = -\frac{V}{\kappa p}$

$\Rightarrow \boxed{\chi = \frac{1}{\kappa p}}$

$c = \sqrt{\frac{1}{\rho \chi}}$

$\rho_k \gg \rho_p$ ← GOSTOTA PLINA; 3 velikostni redi; GOSTOTA KAPLJEVINE

in $\chi_k \lll \chi_p$ (STISLJIVOST KAPLJEVINE; STISLJIVOST PLINA) $\Rightarrow c_k > c_p$ (HITROST V KAPLJEVINI; HITROST (ZVOKA) V PLINU)

Primeri:

$\left.\begin{array}{l} M_z = 29\ \text{kg} \\ T = 0°C \\ p = 1\ \text{bar} \\ \kappa = 1'4 \end{array}\right\} \Rightarrow c = ???$

$\left.\begin{array}{l} \nu = 1000\ \text{Hz} \\ j = 10^{-12}\ \frac{W}{m^2} \end{array}\right\} \Rightarrow \begin{array}{l} u_0 = ??? \\ \Delta p = ??? \end{array}$

ZVOK V 3D (3 dimenzije)

1D: $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$

3D: $\boxed{c^2 \nabla^2 \vec{u} = \frac{\partial^2 \vec{u}}{\partial t^2}}$

RAVNI VAL:

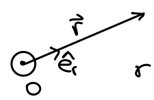
$u(x,t) \longrightarrow \vec{u}(\vec{r},t)$ ↓ odmik od ravnovesne lege (v smeri x)

$\vec{u}(\vec{r},t) = \vec{u}_0 \sin(kx - \omega t + \delta)$; $\vec{z}$ v smeri $\hat{e}_x$ (lahko za smeri y in z)

KROGELNI VALOVANJE

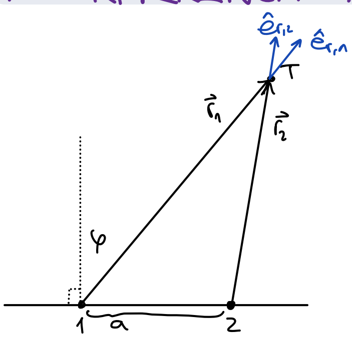
$\vec{u}(\vec{r},t) = u(\vec{r},t)\hat{e}_r$

$u(\vec{r},t) = \frac{u_0}{r}\sin(kr - \omega t + \delta)$



$r = |\vec{r}|$

INTERFERENCA KROGELNEGA VALOVANJA

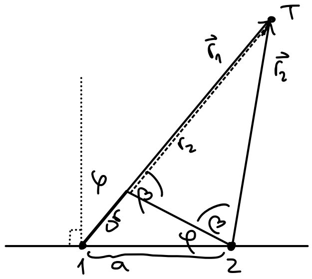


$\vec{u}_1(\vec{r},t) = \frac{u_0}{r_1}\sin(kr_1 - \omega t + \delta_1)\hat{e}_{r_1}$

$\vec{u}_2(\vec{r},t) = \frac{u_0}{r_2}\sin(kr_2 - \omega t + \delta_2)\hat{e}_{r_2}$

$\vec{u} = \vec{u}_1(\vec{r},t) + \vec{u}_2(\vec{r},t)$

Poseben primer: T zelo daleč od zvočnikov: $\boxed{r_1, r_2 \gg a}$ $\quad \hat{e}_{r,1} \doteq \hat{e}_{r,2} = \hat{e}_r$



$\Delta r = r_1 - r_2$

$\boxed{\Delta r \cong a \sin\varphi}$

$\beta \simeq \frac{\pi}{2}$

$r_1 = \Delta r + r_2 =$
$= r_2 + a\sin\varphi =$
$= r_2\left(1 + \frac{a}{r_2}\sin\varphi\right)$

$\rightarrow \frac{1}{r_1} = \frac{1}{r_2\left(1+\frac{a}{r_2}\sin\varphi\right)} =$
$\cong \frac{1}{r_2}\left(1 - \frac{a}{r_2}\sin\varphi\right) =$
$\cong \frac{1}{r_2}$

$$\vec{u} = \frac{u_0}{r_2}\left[\sin(kr_1 - \omega t + \delta_1) + \sin(kr_2 - \omega t + \delta_2)\right]\hat{e}_r$$

$$u = 2\frac{u_0}{r_2}\sin\left(k\frac{r_1+r_2}{2} - \omega t + \frac{\delta_1+\delta_2}{2}\right)\cos\left(k\frac{\Delta r}{2} + \frac{\delta_1-\delta_2}{2}\right)$$

$$\boxed{u = \frac{\tilde{u}_0}{r_2}\sin\left(k\frac{r_1+r_2}{2} - \omega t + \frac{\delta_1+\delta_2}{2}\right) \quad ; \quad \tilde{u}_0 = 2u_0\cos\left(k\frac{\Delta r}{2} + \frac{\delta_1-\delta_2}{2}\right)}$$

Primer: $\delta_1 = \delta_2$ (zvočnika utripata istočasno)

$\Rightarrow \frac{\delta_1-\delta_2}{2} = 0 \quad \Rightarrow \tilde{u}_0 = 2u_0\cos\left(k\frac{\Delta r}{2}\right)$

a) ojačitve: $\underline{\cos\left(k\frac{\Delta r}{2}\right) = \pm 1},$

$k\frac{\Delta r}{2} = n\pi \; ; \; n \in \mathbb{Z}$

$\frac{2\pi}{\lambda}\cdot\frac{a\sin\varphi}{2} = n\pi \quad \Rightarrow \boxed{a\sin\varphi = n\lambda}$ ← pogoj za ojačanje

$\left|\frac{n\lambda}{a}\right| \leq 1$

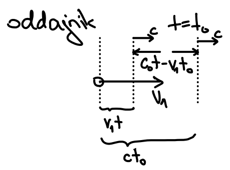
$n=0: \; \sin\varphi = 0 \Rightarrow \varphi = 0$
$\varphi_n = \arcsin\left(\frac{n\lambda}{a}\right)$ } koti, pri katerih so ojačitve

b) oslabitve: $\underline{\cos\left(k\frac{\Delta r}{2}\right) = 0},$

$\rightarrow \boxed{a\sin\varphi = \left(n+\frac{1}{2}\right)\lambda}$

$\varphi_n = \arcsin\left(\frac{\left(n+\frac{1}{2}\right)\lambda}{a}\right)$ ← koti, pri katerih so oslabitve

DOPPLERJEV POJAV



ν... frekvenca oddajnika $\left(t_0 = \frac{1}{\nu}\right)$
λ... valovna dolžina
c... hitrost zvoka v snovi
ν'... frekvenca, ki jo sliši sprejemnik
v_1... hitrost, s kakršno se giblje oddajnik
v_2... hitrost, s kakršno se giblje sprejemnik

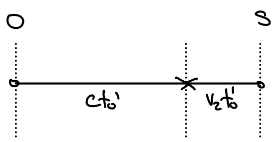
(a) $v_1 = v_2 = 0: \; \nu' = \nu$

(b) $v_2 = 0, \; v_1 \neq 0$: (oddajnik se premika)
$t_1 = 0$... odda 1. valovno čelo
$t = t_0$ ... ct
$\lambda = ct_0 - v_1t_0 =$
$= (c-v_1)t_0 =$
$= \frac{c-v_1}{\nu}$

$\boxed{\nu' = \frac{c}{\lambda} = \frac{c\nu}{c-v_1} = \frac{\nu}{1-\frac{v_1}{c}}}$ $\quad \nu' > \nu \quad$ se približuje

$\boxed{\nu' = \nu\frac{1}{1+\frac{v_1}{c}}}$ $\quad \nu' < \nu \quad$ se oddaljuje

(c) $v_1 = 0, \; v_2 \neq 0$ (sprejemnik se približuje)



$t = 0$
$t = t_0'$: v_2t_0' (razdalja, ki jo prepotuje sprejemnik)
$ct_0' \Rightarrow ct_0' + v_2t_0' = \lambda = \frac{c}{\nu} \rightarrow \frac{1}{\nu'}(c+v_2) = \frac{c}{\nu}$

$\Rightarrow \boxed{\nu' = \nu\left(1+\frac{v_2}{c}\right)}$ se približuje $\qquad \boxed{\nu' = \nu\left(1-\frac{v_2}{c}\right)}$ se oddaljuje

(d) $\boxed{\nu' = \nu\frac{1 \pm \frac{v_2}{c}}{1 \pm \frac{v_1}{c}}}$

2. ELEKTROSTATIKA

ELEKTRIČNA SILA, ELEKTRIČNI NABOJ, ELEKTRIČNO POLJE

$H^+ \underset{r}{\frown} H^+$

masa protona H^+

$m_p = 1{,}7 \cdot 10^{-27}\,kg$ $\quad F_g = \frac{G m_p^2}{r^2} \doteq 2 \cdot 10^{-60}\,N$ ← izvor: m_p, m_p

$G = 6{,}7 \cdot 10^{-11}\,\frac{Nm^2}{kg^2}$

$r = 1\,cm$

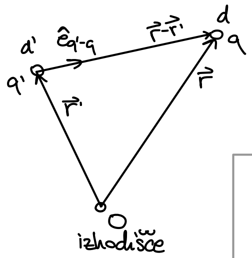
Opazimo: • odbojna sila
• mnogo večja od F_g $\Big\}$ sila $F_{el} \doteq 2 \cdot 10^{-24}\,N$ (ELEKTRIČNA SILA) ← izvor: električni naboj

$\left(\frac{F_{el}}{F_g} \approx 10^{36}\right)$

p, e^-; F_{el} privlačna
q_p: naboj protona
q_e: naboj elektrona $\quad |q_p| = |q_e| = q_0 = 1{,}602176634 \cdot 10^{-19}\,As$ $\quad q_p = +q_0$, $q_e = -q_0$

q_0 ← OSNOVNI NABOJ

Coulomb: $F_{el} \propto \frac{1}{r^2}$
$F_{el} \propto \frac{q'q}{r^2}$



∘ $|\vec{r}-\vec{r}'| \gg d, d'$

∘ $\hat{e}_{q'\to q} = \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|}$

$$\boxed{\vec{F}_{q'\to q} = \frac{q'q}{4\pi\varepsilon_0}\frac{\hat{e}_{q'\to q}}{|\vec{r}-\vec{r}'|^2} = \frac{q'q}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$$ ← COULOMBOV ZAKON

ELEKTRIČNA KONSTANTA
$\varepsilon_0 = 8{,}85 \cdot 10^{-12}\,\frac{(As)^2}{Nm^2} \Big/ \frac{As}{Vm}$

VOLT

$\boxed{V = \frac{Nm}{As}} \Rightarrow \frac{(As)^2}{Nm^2} = \frac{As}{Vm}$

$\Rightarrow \boxed{N = \frac{VAs}{m}}$

$q'q \to m, m'$:

$\vec{F}_{m'\to m} = -Gmm'\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$ ← gravitacijska sila med dvema masama

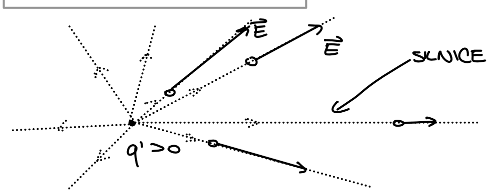
POJEM ELEKTRIČNEGA POLJA:

naboj q': spremeni lastnosti prostora v svoji okolici
⇒ ustvari električno polje

$\boxed{\vec{E}_{q'}(\vec{r}) = \frac{\vec{F}_{q'\to q}}{q}}$ $\quad \left[\frac{N}{As} = \frac{V}{m}\right]$

JAKOST ELEKTRIČNEGA POLJA (v točki z $\vec{r}$, na račun q')

$\Rightarrow \boxed{\vec{E}_{q'}(\vec{r}) = \frac{q'}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$



q_1'; $\vec{r}_1'$
q_2'; $\vec{r}_2'$
⋮
q_m'; $\vec{r}_m'$

$\vec{r}$, $\vec{E}(\vec{r}) = ?$

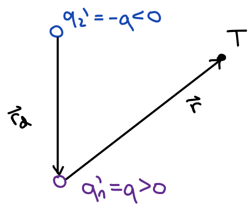
$\left.\begin{array}{l}\vec{F}_{q_1'\to q} = q\vec{E}_{q_1'} \\ \vec{F}_{q_2'\to q} = q\vec{E}_{q_2'} \\ \vdots \\ \vec{F}_{q_m'\to q} = q\vec{E}_{q_m'}\end{array}\right\} \Rightarrow \boxed{\vec{F}_q = \sum_{i=1}^{m} \vec{F}_{q_i'\to q}}$ ← načelo superpozicije

$\vec{E}(\vec{r}) = \frac{\vec{F}_q}{q} \Rightarrow \boxed{\vec{F}_q = q\vec{E}}$

$\Rightarrow q\vec{E} = \sum_{i=1}^{m} q\vec{E}_{q_i'} = q\sum_{i=1}^{m}\vec{E}_{q_i'}$

$\Rightarrow \boxed{\vec{E} = \sum_{i=1}^{m}\vec{E}_{q_i'}}$

ELEKTRIČNO POLJE V OKOLICI ELEKTRIČNEGA DIPOLA:



$$\vec{E}(\vec{r}) = \vec{E}_{q_1'}(\vec{r}) + \vec{E}_{q_2'}(\vec{r})$$

Poseben primer: (T daleč stran od dipola)
$|\vec{r}| = r \gg r_d$

$$\vec{E} \simeq \frac{1}{4\pi\varepsilon_0 r^3}\left[3(\vec{p}_e \cdot \hat{e}_r)\hat{e}_r - \vec{p}_e\right] \qquad \hat{e}_r = \frac{\vec{r}}{r}$$

$\boxed{\vec{p}_e = q\vec{r}_d}$ ← ELEKTRIČNI DIPOLNI MOMENT
$[\mathrm{Asm}]$

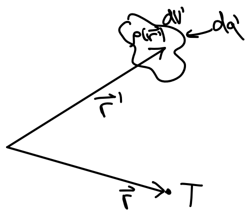
ELEKTRIČNO POLJE V OKOLICI ZVEZNO PORAZDELJENEGA NABOJA

- to je matematični model (v naravi tega ni)

$q = Nq_0,\ N \in \mathbb{Z}$
prosti naboji v naravi (mnogokratniki osnovnega naboja)

$\rho(\vec{r}')$: $dq' = \rho(\vec{r}')dV'$
GOSTOTA ELEKTRIČNEGA NABOJA

$\rightarrow \rho(\vec{r}') = \frac{dq'}{dV'}$

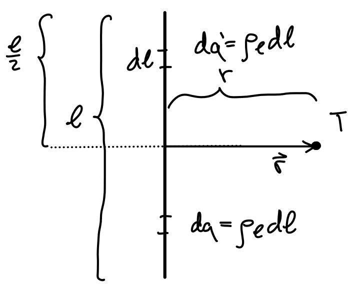


$$d\vec{E} = \frac{dq'}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \frac{\rho(\vec{r}')}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dV'$$

$$\Rightarrow \boxed{\vec{E}(\vec{r}) = \int_{\mathbb{R}^3} \frac{\rho(\vec{r}')}{4\pi\varepsilon_0}\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}dV'}$$

PRIMERI:

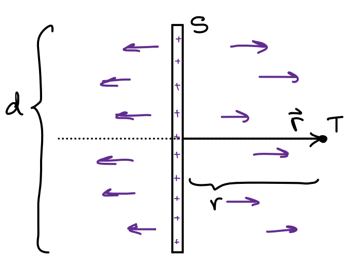
(1) $\vec{E}$ na simetrali zelo dolge ravne enakomerno nabite žice



ρ_e ... dolžinska gostota naboja
$\rho_e = \frac{q'}{\ell} = \mathrm{konst.}$

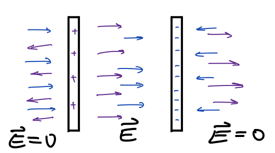
$\boxed{\vec{E}(\vec{r}) = \frac{\rho_e}{2\pi\varepsilon_0 r}\hat{e}_r}$; $\hat{e}_r = \frac{\vec{r}}{r}$

(2) $\vec{E}$ na simetrali velike enakomerno nabite okrogle plošče



$d \gg r \qquad \rho_S = \frac{q}{S} = \mathrm{konst.}$

$\boxed{\vec{E} = \frac{\rho_S}{2\varepsilon_0}\hat{e}_r}$ ← neodvisno od oddaljenosti (r)
⇒ $\vec{E}$ blizu plošče homogeno

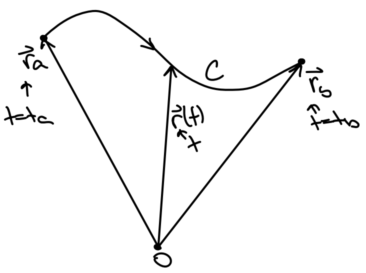


← KONDENZATOR

$\boxed{\vec{E} = \frac{|\rho_S|}{\varepsilon_0}}$ $\qquad \hat{e}_E$: od + proti −

ELEKTRIČNA NAPETOST, ELEKTRIČNI POTENCIAL, DELO ELEKTRIČNE SILE

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\mathbb{R}^3} \rho(\vec{r}\,')\,\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\,dV'$$



ELEKTRIČNA NAPETOST med $\vec{r}_a$ in $\vec{r}_b$ po C

$$\boxed{U(\vec{r}_a \to \vec{r}_b; C) = -\int_C \vec{E}\,d\vec{s}} \quad [V]$$

$$\boxed{\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\mathbb{R}^3} \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,dV'} \quad \left[\frac{AsVm\,m^3}{m^3 Asm} = V\right]$$

ELEKTRIČNI POTENCIAL v točki $\vec{r}$

Zveza med $\vec{E}$ in Φ:

$\nabla\Phi$ – predpostavimo, da ∇ in $\int_{\mathbb{R}^3} dV'$ lahko zamenjamo

$$\nabla\Phi = \frac{1}{4\pi\varepsilon_0}\nabla\int_{\mathbb{R}^3}\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}dV' = \frac{1}{4\pi\varepsilon_0}\int_{\mathbb{R}^3}\nabla\left(\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\right)dV' = \frac{1}{4\pi\varepsilon_0}\int_{\mathbb{R}^3}\rho(\vec{r}\,')\,\nabla\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right)dV' = -\vec{E}$$

$$\nabla\left(\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = \left(\partial_x(\dots), \partial_y(\dots), \partial_z(\dots)\right) = \left((x-x'),(y-y'),(z-z')\right)\frac{-1}{|\vec{r}-\vec{r}\,'|^3} = -\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

$$\partial_x(\dots) = -\frac{1}{2}\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-\frac{3}{2}}2(x-x') = \frac{(x-x')}{|\vec{r}-\vec{r}\,'|^3}$$

(simetrično ∂_y in ∂_z)

$\Rightarrow \boxed{\vec{E} = -\nabla\Phi}$ ← velja v elektrostatskem polju, ne v splošnem

$\Rightarrow \boxed{\nabla\times\vec{E} = 0}$ ← IZREK O ELEKTRIČNI NAPETOSTI V DIFERENCIALNI OBLIKI $\quad (\nabla\times\nabla\Phi = 0)$

rotor

Posledice:

(a) $U(\vec{r}_a \to \vec{r}_b, C)$ neodvisna od C

$$\nabla\Phi\,\dot{\vec{r}} = \dot{\Phi} = \frac{\partial\Phi}{\partial x}\dot{x} + \frac{\partial\Phi}{\partial y}\dot{y} + \frac{\partial\Phi}{\partial z}\dot{z} = \begin{bmatrix}\partial_x\Phi\\ \partial_y\Phi\\ \partial_z\Phi\end{bmatrix}\cdot\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix}$$

$$\| \qquad -\int_S \vec{E}\,d\vec{s} = -\int_{t_a}^{t_b}\vec{E}\cdot\dot{\vec{r}}\,dt = -\int_{t_a}^{t_b} -\nabla\Phi\,\dot{\vec{r}}\,dt = \int_{t_a}^{t_b}\dot{\Phi}\,dt = \boxed{\Phi(t_b)-\Phi(t_a) = U(\vec{r}_a\to\vec{r}_b)}$$

(b) $\partial S = C$ ... zaključena pot:

$$-\oint_{\partial S}\vec{E}\,d\vec{s} = \Phi(\vec{r}_b) - \Phi(\vec{r}_a) = 0$$

$\Rightarrow \boxed{\oint_{\partial S}\vec{E}\,d\vec{s} = 0}$ ← IZREK O ELEKTRIČNI NAPETOSTI V INTEGRALSKI OBLIKI

Stokesov izrek: ∂S: gladka orientabilna ploskev, E_x, E_y, E_z

$$\oint_{\partial S}\vec{E}\,d\vec{s} = \int_S(\nabla\times\vec{E})\,d\vec{S} = \int_S \vec{0}\,dS = 0$$

q testni naboj v $\vec{E}$: $\qquad \vec{F}$ konservativna $\qquad \boxed{W_{ep} = q\Phi}$

$$\boxed{A_{el}} = \int_C \vec{F}\,d\vec{s} = \int_C q\vec{E}\,d\vec{s} = q\int_C \vec{E}\,d\vec{s} = -qU(\vec{r}_a\to\vec{r}_b) = -q\left[\Phi(\vec{r}_b)-\Phi(\vec{r}_a)\right] = \boxed{-\Delta W_{ep}}$$

DELO ELEKTRIČNE SILE

Primer: napetost U med ploščama kondenzatorja:

$\vec{r}_a = \begin{bmatrix}0\\0\\0\end{bmatrix}$, $\oplus \leftarrow$ središče pozitivno nabite plošče $\qquad \vec{r}_b = \begin{bmatrix}\ell\\0\\0\end{bmatrix}$, $\ominus \leftarrow$ središče negativno nabite plošče

$$\vec{r} = \begin{bmatrix}x\\y\\z\end{bmatrix}; \quad \dot{\vec{r}} = \begin{bmatrix}\dot{x}\\\dot{y}\\\dot{z}\end{bmatrix}$$

$$U(\vec{r}_a \to \vec{r}_b) = -\int_C \vec{E}\, d\vec{s} = -\int_{t_a}^{t_b} \vec{E}\,\dot{\vec{r}}\,dt = -\int_{t_a}^{t_b} \frac{\beta_s}{\varepsilon_0}\hat{e}_x \cdot \dot{\vec{r}}\, dt = -\int_{t_a}^{t_b} \frac{q}{S\varepsilon_0}\dot{x}\,dt = -\frac{q}{S\varepsilon_0}\int_{t_a}^{t_b} \dot{x}\,dt =$$

$$= -\frac{q}{S\varepsilon_0}\left(x|_{t=t_b} - x|_{t=t_a}\right) = -\frac{q\ell}{S\varepsilon_0}$$

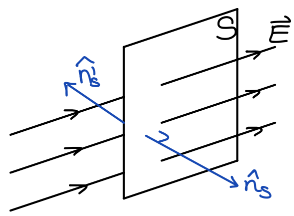
$$\Rightarrow |U| = \frac{|q|\ell}{S\varepsilon_0}$$

$$\to \boxed{C = \frac{|q|}{|U|} = \frac{\varepsilon_0 S}{\ell}} \quad \left[F = \frac{As}{V}\right]$$

KAPACITETA KONDENZATORJA

PRETOK ELEKTRIČNEGA POLJA, Φ_E (ELEKTRIČNI PRETOK)

HOMOGENO ELEKTRIČNO POLJE $\vec{E}$, ravna ploskev:



$\vec{S} = S \cdot \hat{n}_S$

Intuitivno: štejemo silnice, ki prebadajo ploskev

Formalno: $\boxed{\Phi_E = \varepsilon_0 \vec{E}\cdot\vec{S} = \varepsilon_0 \vec{E}\cdot S\hat{n}_S}$ $\left[\frac{As}{Vm\,m}\frac{V\,m^2}{} = As\right]$

SPLOŠNA DEFINICIJA: Φ_E (polje ne nujno homogeno in ravno)

$\vec{S} \to d\vec{S} = \hat{n}_{dS}\, dS$ ~ približna ravna majhna ploskev

$$d\Phi_E = \varepsilon_0 \vec{E}\cdot d\vec{S}$$

$$\Rightarrow \boxed{\Phi_E = \int_S \varepsilon_0 \vec{E}\, d\vec{S} = \varepsilon_0 \int_S \vec{E}\cdot\vec{n}_{dS}\, dS}$$

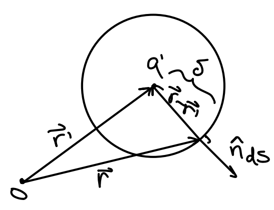
$\vec{E}\cdot\vec{S} = ES\cos\beta$, $\beta = \sphericalangle ES$

Komentar: a) Φ_E je skalar (nima smeri), ima pa predznak: $\operatorname{sgn}\Phi_E = \begin{cases}1; & \beta \text{ oster}\\ -1; & \beta \text{ top}\end{cases}$

b) Predznak Φ_E odvisen od izbire $\hat{n}_S$ ali $\hat{n}_S' = -\hat{n}_S$.

Primer: krogla $K(\vec{r}', \delta)$ in sfera ∂K: $S = \int_{\partial K} dS = 4\pi\delta^2$

$\hat{n}_{dS}$ zunanja normala



$$\hat{n}_{dS} = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$$

$$\vec{E}_{q'}(\vec{r}) = \frac{q'}{4\pi\varepsilon_0}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$\Rightarrow \vec{E}\,d\vec{S} = \vec{E}\hat{n}_{dS}\,dS = \frac{q'}{4\pi\varepsilon_0}\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\cdot\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}\,dS = \frac{q'}{4\pi\varepsilon_0}\frac{1}{|\vec{r}-\vec{r}'|^2}\,dS = \frac{q'}{4\pi\varepsilon_0\delta^2}\,dS$$

$$\Rightarrow \boxed{\Phi_E = \frac{\varepsilon_0 q'}{4\pi\varepsilon_0\delta^2}\int_{\partial K} dS = \frac{q'}{4\pi\delta^2}4\pi\delta^2 = q'}$$

IZREK O ELEKTRIČNEM PRETOKU: $\boxed{\Phi_E(\partial V) = \int_V \rho \, dV}$ zaobjeti naboj

zaključena ploskev

$$\boxed{\int_{\partial V} \varepsilon_0 \vec{E} \, d\vec{S} = \int_V \rho \, dV}$$

IZREK O ELEKTRIČNEM PRETOKU V DIFERENCIALNI OBLIKI

Gauss: $\int_{\partial V} \varepsilon_0 \vec{E} d\vec{S} = \int_V \varepsilon_0 \nabla \cdot \vec{E} \, dV = \int_V \rho \, dV \Rightarrow \varepsilon_0 \nabla \vec{E} = \rho \rightarrow \boxed{\nabla \vec{E} = \frac{\rho}{\varepsilon_0}}$

REORGANIZACIJA ("obrnjena logika")

Izrek (Helmholtz): $\vec{E}$ v $\mathbb{R}^3$, $r \to \infty$, $\vec{E}$ vsaj $\frac{1}{r}$:

$$\Rightarrow \boxed{\vec{E} = -\nabla \Phi + \nabla \times \vec{A}}$$

$$\Phi = \frac{1}{4\pi} \int_{\mathbb{R}^3} \frac{\nabla' \cdot \vec{E}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV' \qquad \left(\nabla' = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'}\right)\right)$$

$$\vec{A} = \frac{1}{4\pi} \int_{\mathbb{R}^3} \frac{\nabla' \times \vec{E}(\vec{r}')}{|\vec{r} - \vec{r}'|^3} dV'$$

Električno polje $\vec{E}$: $\boxed{\nabla \times \vec{E}(\vec{r}) = 0}$ $\Rightarrow \nabla' \times \vec{E}(\vec{r}') = 0$

Izrek: $\boxed{\nabla \vec{E} = \frac{\rho}{\varepsilon_0}}$

$$\Rightarrow \vec{E} = -\nabla \left[\frac{1}{4\pi} \int_{\mathbb{R}^3} \frac{\nabla' \times E(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'\right] \underbrace{(+ \nabla \times \overset{0}{\vec{A}})}_{0}$$

$$= -\nabla \left[\frac{1}{4\pi} \int_{\mathbb{R}^3} \frac{\rho(\vec{r}')}{\varepsilon_0} \frac{1}{|\vec{r} - \vec{r}'|} dV'\right]$$

$$= -\int_{\mathbb{R}^3} \frac{\rho(\vec{r}')}{4\pi\varepsilon_0} \nabla \left[\frac{1}{|\vec{r} - \vec{r}'|}\right] dV' = \boxed{\int_{\mathbb{R}^3} \frac{\rho(\vec{r}')}{4\pi\varepsilon_0} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dV'} \; *$$

COULOMBOV IZREK (zakon)

(Do sedaj: Coulombov zakon $\xrightarrow{\vec{E} \text{ def.}} \vec{F}_{q' \to q} = q \vec{E}_{q'} \xrightarrow{\text{superpozicija}} \vec{E}_{\rho(\vec{r}')}(\vec{r}) = \int_{\mathbb{R}^3} \frac{\rho(\vec{r}')}{4\pi\varepsilon_0} \frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dV'$ —...

... $\xrightarrow{\text{izreka}}$ $\nabla \times \vec{E} = 0$ in $\nabla \vec{E} = \frac{\rho}{\varepsilon_0}$)

Sedaj: $\underbrace{\nabla \times \vec{E} = 0; \; \nabla \vec{E} = \frac{\rho}{\varepsilon_0}, \; \vec{F} = q\vec{E}}_{\text{FIZIKALNI ZAKONI}} \xrightarrow{\text{izrek}} \underbrace{\overset{*}{\vec{E}}(\vec{r}) \cdots}_{\text{COULOMBOV IZREK}}$

Komentar: $\nabla \vec{E} = \frac{\rho}{\varepsilon_0}$ tudi velja za premikajoče naboje (izven polja)

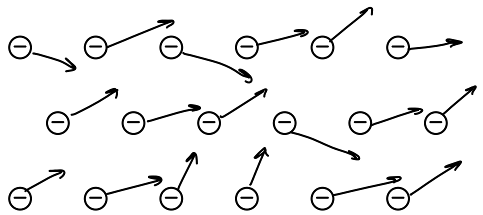
$\nabla \times \vec{E} = 0$ res, če nimamo spremenljivih magnetnih polj: $\frac{\partial \vec{B}}{\partial t} = 0$

Komentar: $\vec{F} = q\vec{E}$ res tudi, če $\vec{v}_q \neq 0$

3. ELEKTRIČNI TOK

Opazovali bomo gibanje nabojev pod vplivom (zunanjega) električnega polja.

GOSTOTA ELEKTRIČNEGA TOKA



število prostogibljivih nabojev

$$\boxed{n \equiv \frac{N}{V}} \quad \left[\frac{1}{m^3}\right]$$

q_1 [As] ← naboj enega nosilca

$$\Rightarrow \boxed{\rho_e \equiv q_1 n} \quad \left[\frac{As}{m^3}\right]$$

← številska gostota gibljivih nabojev

$$\boxed{\vec{v} = \frac{1}{N}\sum_i \vec{v}_i}$$

$$\Rightarrow \boxed{\vec{j}_e \equiv \rho_e \vec{v}}$$

← GOSTOTA ELEKTRIČNEGA TOKA $\left[\frac{A}{m^2}\right]$

$$v \equiv |\vec{v}| > 0 \; ; \quad \hat{e}_v \equiv \frac{\vec{v}}{v} \Rightarrow \vec{v} = v \cdot \hat{e}_v$$

$$\rightarrow \boxed{\vec{j}_e = \rho_e v \hat{e}_v = q_1 n v \hat{e}_v}$$

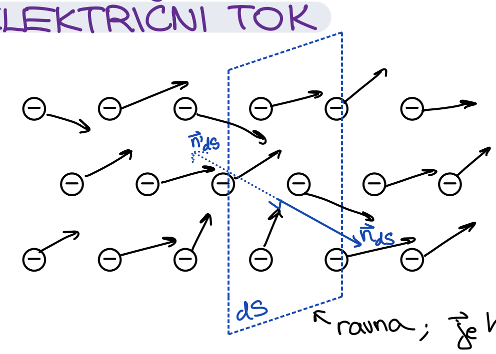
$$j_e \equiv |\vec{j}_e| > 0, \quad \hat{e}_{j_e} = \frac{\vec{j}_e}{j_e} \Rightarrow \boxed{\vec{j}_e = j_e \hat{e}_j}$$

↑ smer gostote električnega toka (smer električnega toka)

$$j = |\vec{j}| = |\rho_e \vec{v}| = |\rho_e| \quad \Rightarrow \boxed{\vec{j}_e = |\rho_e| v \hat{e}_j}$$

$$\rightarrow \rho_e v \hat{e}_v = |\rho_e| v \hat{e}_j \quad \Rightarrow \boxed{\hat{e}_j = \hat{e}_v \frac{\rho_e}{|\rho_e|}} = \begin{cases} +\hat{e}_v \; ; & \rho_e > 0 \\ -\hat{e}_v \; ; & \rho_e < 0 \end{cases}$$

ELEKTRIČNI TOK



↖ ravna; $\vec{j}_e$ homogeno

ELEKTRIČNI TOK SKOZI dS

$$\Rightarrow \boxed{dI \equiv \vec{j}_e \cdot d\vec{S}} \quad [A]$$

Splošna definicija: (S ukrivljena, $\vec{j}_e$ nehomogeno)

$$\boxed{I \equiv \int dI = \int_S \vec{j}_e \, d\vec{S}} \quad [A]$$

↖ pretok

Analogija: $\vec{j}_e \leftrightarrow \varepsilon_0 \vec{E}$

$$I = \int_S \vec{j}_e d\vec{S} \leftrightarrow \Phi_E \int_S \varepsilon_0 \vec{E} d\vec{S}$$

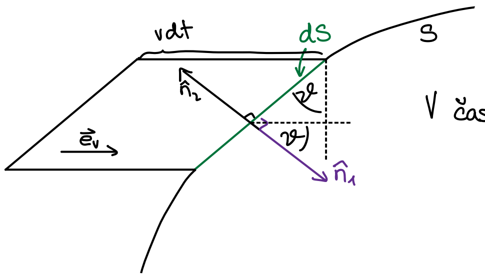
⇒ I nima smeri

⇒ predznak odvisen od izbire orientacije ploskve ($\hat{n}_S$ ali $-\hat{n}_S$)

Primer: homogeno $\vec{j}_e$, ravna S ($\hat{n}_{dS} = \hat{n}_S$)

$$I = \int_S \vec{j}_e d\vec{S} = \int_S j_e \hat{e}_j \hat{n}_{dS} \, dS = j_e \hat{e}_j \hat{n}_S \int_S dS = j_e \hat{e}_j \hat{n}_S S = \vec{j}_e \vec{S}$$

INTERPRETACIJA ELEKTRIČNEGA TOKA



$I = \text{pretok}(\vec{j})$

V času dt: $d^3V = v\,dt\,d^2S\cos\vartheta\ ;\ \cos\vartheta = \hat{e}_v\cdot\hat{n}_1$

$= v\cdot dt\cdot d^2S\,\hat{e}_v\cdot\hat{n}_1$

$d^3N = n\cdot d^3V = dt\,n\,v\,\hat{e}_v\cdot\hat{n}_1\,d^2S$

$d^3q = q_1\,d^3N = dt\,q_1 n v\,\hat{e}_v\hat{n}_1\,d^2S$

$= dt\,\vec{j}_e\,\hat{n}_1\,d^2S$

$= dt\,\vec{j}_e\,d^2\vec{S}_1$

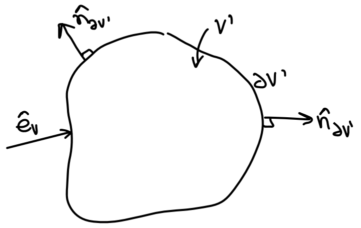
$\Rightarrow dq = dt\underbrace{\int \vec{j}_e\,d^2\vec{S}_n}_{I_{\hat{n}_1}} \Rightarrow \boxed{I_{\hat{n}_1} = \frac{dq}{dt}}\ ;\ \hat{n}_1\hat{e}_v > 0$

$\boxed{I_{\hat{n}_2} = -\frac{dq}{dt}}\ ;\ \hat{n}_2\hat{e}_v < 0$

$$\Rightarrow \boxed{dq = \begin{cases} I\,dt\ ; & \hat{n}\hat{e}_v > 0 \\ -I\,dt\ ; & \hat{n}\hat{e}_v < 0 \end{cases}}$$

ZAKON O OHRANITVI ELEKTRIČNEGA NABOJA, KONTINUITETNA ENAČBA in PRVI KIRCHOFFOV IZREK

a)



$\hat{e}_v$ v V' (naboj pritekajo v V')

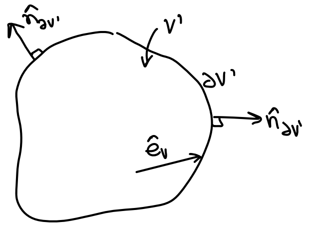
$\Rightarrow \hat{e}_v\cdot\hat{n}_{\partial V'} < 0$

$\Rightarrow \boxed{dq = -I\,dt}$

$dq'_{\partial V'}$ ... sprememba naboja v V' zaradi pretakanja skozi $\partial V'$

$\boxed{dq'_{\partial V'} = dq = -I\,dt}$

b)



$\hat{e}_v$ iz V' (naboji tečejo iz V')

$\Rightarrow \hat{e}_v\,\hat{n}_{\partial V'} > 0$

$\Rightarrow \boxed{dq = I\,dt}$

$\Rightarrow \boxed{dq'_{\partial V'} = -dq = -I\,dt}$

V splošnem: $\boxed{dq'_{\partial V'} = -dt\oiint_{\partial V}\vec{j}_e\,d\vec{S} = -dt\iiint_V \nabla\vec{j}\,dV}$

q': celoten naboj v V'

$q' = \iiint_{V'} \rho_e\,dV$

$\frac{\partial q'}{\partial t}$: sprememba naboja v (fiksnem) V' na enoto časa

$\frac{\partial q'}{\partial t} = \frac{\partial}{\partial t}\iiint_{V'} \rho(\vec{r})\,dV = \iiint_{V'} \frac{\partial}{\partial t}\rho\,dV$

dq': sprememba naboja v V' v intervalu dt

$\boxed{dq' = \frac{\partial q'}{\partial t}dt = dt\iiint_{V'} \frac{\partial\rho}{\partial t}\,dV}$

ZAKON O OHRANITVI ELEKTRIČNEGA NABOJA (fizikalni zakon, eksperimentalno dejstvo)

naboja ne moremo ne ustvariti ne izničiti

$\boxed{dq' = dq'_{\partial V'}}$

$$\Rightarrow dt \iiint_{V'} \frac{\partial \rho}{\partial t} dV = -dt \iiint_{V'} \nabla \vec{j}\, dV$$

$$dt \iiint_{V'} \frac{\partial \rho}{\partial t} dV = \iiint_{V'} -\nabla \vec{j}\, dV \Rightarrow \iiint_{V'} \left(\frac{\partial \rho}{\partial t} + \nabla \vec{j}\right) dV = 0, \quad \forall V'$$

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} = -\nabla \vec{j}}$$ ← KONTINUITETNA ENAČBA

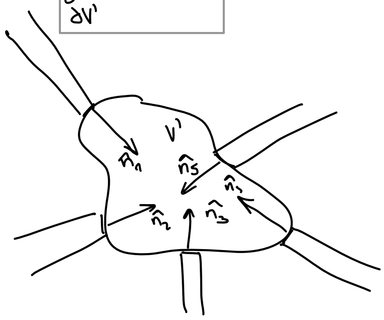
(diferencialna oblika)

$$\iiint_{V'} \frac{\partial \rho}{\partial t} dV = \boxed{-\oint_{\partial V'} \vec{j}\, d\vec{S} = \frac{\partial q'}{\partial t}}$$ (integralna oblika)

Poseben primer: $\boxed{\frac{\partial q}{\partial t} = 0}$

$$\Rightarrow \nabla \vec{j} = 0$$
$$\Rightarrow \iiint_{V'} \nabla \vec{j} = 0 \Rightarrow \mp \oint_{\partial V'} \vec{j}\, d\vec{S} = 0$$

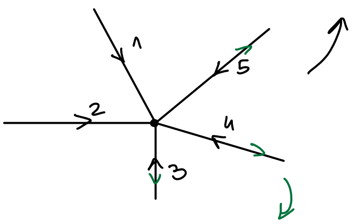
$\Rightarrow \boxed{\oint_{\partial V'} \vec{j}\, d\vec{S} = 0}$; $\hat{n}_{\partial V'}$ vsi kažejo v isto smer ; npr v V'



$$I_i = \int_{\partial V'_i} \vec{j}\, dS$$

$$\oint_{\partial V'} \vec{j}\, d\vec{S} = \sum_{i=1}^{5} \int_{\partial V'_i} \vec{j}\, d\vec{S} = \sum_{i=1}^{5} I_i$$

$\boxed{\sum_i I_i = 0}$ ← 1. KIRCHOFFOV IZREK



$I_3' = -I_3,\ I_4' = -I_4,\ I_5' = -I_5$: $\sum_1^5 I_i = \sum_1^2 I_i - \sum_3^5 I_i \overset{j=i-2}{=} \sum_1^2 I_i - \sum_1^3 I_j' = 0$

$$\Rightarrow \boxed{\sum_1^2 I_i = \sum_1^3 I_j'}$$

OHMOV ZAKON

$\boxed{\vec{j} = \frac{1}{\xi}\vec{E}}$; ξ: specifična upornost snovi (pravilna oznaka ς)

$\left[\frac{V}{m}\frac{m^2}{A} = \frac{Vm}{A} = \Omega m = \Omega \frac{mm^2}{m}\right]$ ohm

$\vec{j} \propto \vec{E}$

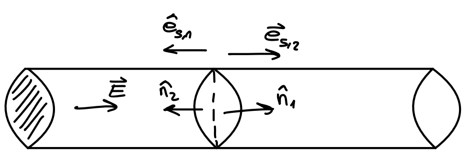
material	$\xi\ [\frac{\Omega mm^2}{m}]$
Cu	0,0172
Fe	0,1
slana voda	$8\cdot10^4$
voda	$3\cdot10^7$
steklo	10^{16}
SiO_2	10^{23}

↑ PREVODNIKI

↓ IZOLATORJI

∘ linearni zakon upora: Drude

MAKROSKOPSKA SLIKA:
$U, I, \xi \rightarrow R$
zanima nas povezava



$\xi = $ konst.

$\vec{j} = \frac{1}{\xi}\vec{E} \rightarrow \vec{E} = \xi\vec{j}$

$\hat{n}_1$ ali $\hat{n}_2$: $I_{1,2} = \vec{j}\cdot\vec{S} = \vec{j}\,\hat{n}_{1,2} S$

$\vec{e}_{s,1}$ ali $\vec{e}_{s,2}$: $U_{1,2} = -\int \vec{E}\, d\vec{S}_{1,2} = -\vec{E}\,\vec{e}_{s,1/2}\, l$

(a) $\hat{e}_s = +\hat{n}$ $\quad(\hat{e}_s = \hat{e}_{s,1}$ in $\hat{n} = \hat{n}_1$ ali $\hat{e}_s = \hat{e}_{s,2}$ in $\hat{n} = \hat{n}_2)$

$$U = -\vec{E}\hat{e}_s \ell = -\vec{E}\hat{n} = -\xi \vec{j}\,\hat{n}\,\ell = -\frac{\xi}{S}\ell I = -RI\,; \quad \boxed{R = \frac{\xi \ell}{S}} \quad [\Omega]$$

(b) $\hat{e}_s = -\hat{n}$

$$\Rightarrow U = -\vec{E}\hat{e}_s \ell = -\vec{E}\cdot(-\hat{n})\ell = \vec{E}\hat{n}\ell = \xi \vec{j}\,\hat{n}\,\ell = \xi \ell \frac{I}{S} = RI$$

$$|I| = jS = n v q_0 S\,; \quad n = \frac{N}{V} = \frac{1}{V}\frac{m}{M}\underbrace{N_A}_{\text{avogadrovo število}} = \frac{\rho N_A}{M}$$

Primer:

baker (Cu): $S = 1\ mm^2 = 10^{-6}\ m^2$

$I = 1\ A$

$\rho_{Cu} = 9\cdot 10^3\ \frac{kg}{m^3}$

$M_{Cu} = 63{,}5\ \frac{kg}{kmol}$

$N_A = 6\cdot 10^{26}\ kmol^{-1}$

$v = ?$

$$v = \frac{|I|}{nSq_0} = \frac{|I|\cdot M_{Cu}}{\rho_{Cu}\cdot N_A\cdot S q_0} =$$

$$= \frac{1\,A\cdot 63{,}5\ kg\ \ m^3\ kmol}{kmol\ \ 9\cdot 10^3\ kg\cdot 6\cdot 10^{26}\cdot 10^{-6}\,m^2\cdot 1{,}6\cdot 10^{-19}\,As} =$$

$$= \frac{63{,}5}{9\cdot 6\cdot 1{,}6}\cdot 10^{-4}\ \frac{m}{s} \approx 0{,}1\ \frac{mm}{s}$$

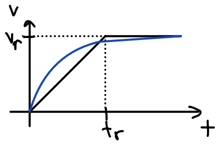
Primer:

Cu, $v = 0 \rightarrow v_r (= 0{,}1\ \frac{mm}{s})$

$t_r = ?$

$$v_r = \frac{|I|\cdot M_{Cu}}{\rho_{Cu} N_A S q_0} = \frac{|U|\cdot M_{Cu}}{R \rho_{Cu} N_A S q_0} = \frac{|U|\cdot S\cdot M_{Cu}}{\xi \ell \rho_{Cu} N_A S q_0}$$

Poenostavitev: $a = $ konst.



$$v_r = a t_r \Rightarrow t_r = \frac{v_r}{a}$$

$$a = \frac{F_{el}}{m_1} = \frac{q_0 E}{m_1} = \frac{q_0 |U|}{m_1 \ell} = \frac{q_0 |U| N_A}{\ell M_{Cu} r} \qquad (m_1 = \frac{M_e}{M_n} r)$$

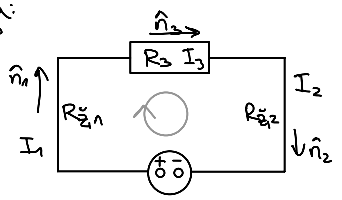
$$\rightarrow t_r = \frac{|U| M_{Cu}^2 \ell r}{\xi \ell \rho_{Cu} N_A q_0^2 |U| N_A} = \frac{M_{Cu}^2 r}{\xi \rho_{Cu} N_A^2 q_0^2} \sim ps \text{ (piko sekunda)}$$

DRUGI KIRCHHOFFOV ZAKON

- zakon o električni napetosti: $\boxed{-\oint_C \vec{E}\,d\vec{s} = 0}$
- omejena veljavnost: $\frac{\partial \vec{B}}{\partial t} = 0$

Primer vezja:

ZAPOREDNA VEZAVA:



izberemo: a) smer obhoda $\hat{e}_s$ \
b) smeri $\hat{n}_1, \hat{n}_2, \hat{n}_3$ $\}$ $\hat{e}_{s,i} = \hat{n}_i$ (v našem primeru)

$$-\oint_C \vec{E}\,d\vec{s} = -\int_{C_1}\vec{E}\,d\vec{s} - \int_{C_3}\vec{E}\,d\vec{s} - \int_{C_2}\vec{E}\,d\vec{s} - \int_{C_g}\vec{E}\,d\vec{s} = 0$$

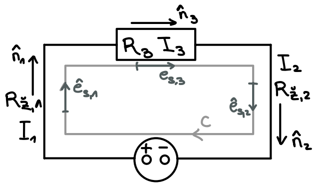
C_g ← generator

$$-\int_{C_g}\vec{E}\,d\vec{s} = U_g \begin{cases} >0: & - \rightarrow + \\ <0: & + \rightarrow - \end{cases} \qquad (\text{naša pot: } - \rightarrow +)$$

$$-\int_{C_1}\vec{E}\,d\vec{s} = U_{ž,1} = -I_1 R_1 \qquad -\int_{C_2}\vec{E}\,d\vec{s} = U_{ž,2} = -I_2 R_2 \qquad (\text{"}-\text{" ker sta } \hat{e}_{1/2} \text{ in } \hat{n}_{1/2} \text{ v isti smeri})$$

$$-\int_{C_3}\vec{E}\,d\vec{s} = U_{R_3} = -I_3 R_3$$

1. Kirchoffov izrek $\Rightarrow I_1 = I_2 = I_3 = I\,;\ U_{ž1} + U_{ž2} + U_{R_3} + U_g = 0 \Rightarrow \boxed{\sum_i U_i = 0}$



$$\left.\begin{aligned}\hat{e}_{s,1}&=\hat{n}_1\\ \hat{e}_{s,2}&=\hat{n}_2\\ \hat{e}_{s,3}&=\hat{n}_3\end{aligned}\right\}\Rightarrow \begin{aligned}U_{\check{z},1}&=-I R_{\check{z},1}\\ U_{\check{z},2}&=-I R_{\check{z},2}\\ U_{R_3}&=-I R_3\end{aligned}$$

$U_g + U_{\check{z},1} + U_{\check{z},2} + U_{R_3} = 0$; $U_g > 0$ (skozi generator od $-$ k $+$)

$\Rightarrow U_g = |U_g|$

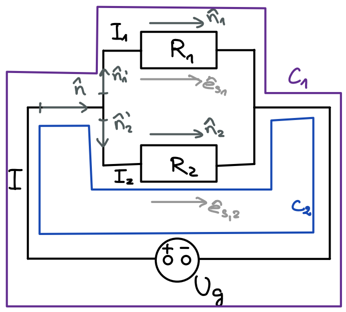
$|U_g| - IR_{\check{z},1} - IR_{\check{z},2} - IR_3 = 0$

$|U_g| = I\underbrace{(R_{\check{z},1} + R_{\check{z},2} + R_3)}_{\overset{\shortparallel}{R}}$ ← zaporedna vezava

R ← nadomestni upor

$\rightarrow \boxed{I = \dfrac{|U_g|}{R}}$

VZPOREDNA VEZAVA:



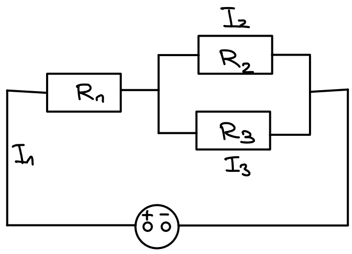
$I = I_1 + I_2$; $R_{\check{z}} \ll R_1, R_2$

$$\left.\begin{aligned}\hat{e}_{s,1}&=\hat{n}_1\\ \hat{e}_{s,2}&=\hat{n}_2\end{aligned}\right\}\Rightarrow \begin{aligned}U_1&=-I_1R_1\\ U_2&=-I_2R_2\end{aligned}$$

2. Kirchhoffov izrek: $|U_g| - I_1R_1 = 0 \Rightarrow$; $|U_g| - I_2R_2 = 0 \Rightarrow$ $\boxed{\begin{aligned}I_1&=\frac{|U_g|}{R_1}\\ I_2&=\frac{|U_g|}{R_2}\end{aligned}}$

1. Kirchhoffov izrek: $I = \dfrac{|U_g|}{R_1} + \dfrac{|U_g|}{R_2} = |U_g|\left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

$\Rightarrow \dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ ← vzporedna vezava $\rightarrow R = \dfrac{R_1R_2}{R_1+R_2}$

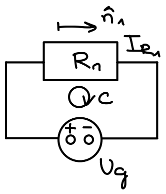


↙ Podobno velja za bolj sestavljene vezave.

MERITEV TOKA

—(A)— ← ampermeter (merilec toka) ; upornost R_A (čim manjša)

(a)

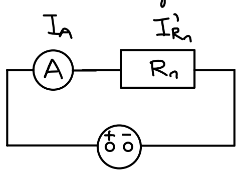


$\hat{n}_1 = \hat{e}_s \Rightarrow U_{R_1} = -I_1R_1$

$R_{\check{z}} \ll R_1$

$|U_g| + U_{R_1} = 0 \rightarrow \boxed{I_1 = \dfrac{|U_g|}{R_1}}$

(b)



$I_A = I_{R_1}'$; v splošnem: $I_A = I_{R_1}' \neq I_{R_1}$

$R = R_A + R_1 \Rightarrow I_A = \dfrac{|U_g|}{R} = \dfrac{|U_g|}{R_1 + R_A} \neq I_{R_1}$

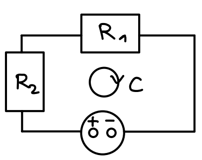
$R_A \ll R_1$: $I_A = \dfrac{|U_g|}{R_1\left(1 + \frac{R_A}{R_1}\right)} \cong \dfrac{|U_g|}{R_1}\left(1 - \dfrac{R_A}{R_1}\right) \cong \dfrac{|U_g|}{R_1} = I_{R_1}$

MERITEV NAPETOSTI

—(V)— ← VOLTMETER = ampermeter z znanim uporom, R_V: $U_V = \pm I_V R_V$

↑ čim večja

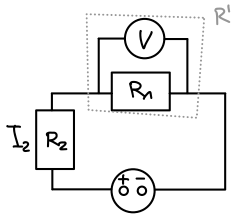
(a)



$R_{\check{z}} \ll R_1, R_2$

$U_{R_1} = -I_1R_1 = -\dfrac{|U_g|}{R}\cdot R_1 = -|U_g|\dfrac{R_1}{R_1+R_2}$

(b)



Ⓥ in R_1 vezana vzporedno

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_V} \rightarrow R' = \frac{R_1 R_V}{R_1 + R_V}$$

$$R = R_2 + R' = R_2 + \frac{R_1 R_V}{R_1 + R_V} \qquad I_2 = \frac{|U_g|}{R}$$

zaporedno vezano

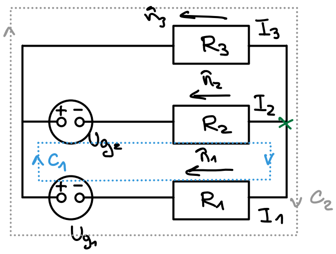
$\neq U_{R_1}$

2. Kirchhoffov izrek: $|U_g| - I_2 R_2 - \underbrace{I_V R_V}_{U_V} = 0 \Rightarrow \boxed{U_V = I_2 R_2 - |U_g| = \frac{|U_g|}{R} R_2 - |U_g| = |U_g|\left(\frac{R_2}{R} - 1\right)}$

$$R_V \gg R_1: \quad R' = \frac{R_1 R_V}{R_V + R_1} = \frac{R_1 R_V}{R_V\left(1 + \frac{R_1}{R_V}\right)} \simeq R_1\left(1 - \frac{R_1}{R_V}\right) \sim R_1 \quad \Rightarrow R = R_2 + R' \simeq R_2 + R_1$$

$$\Rightarrow U_V = -|U_g|\left(1 - \frac{R_2}{R_1 + R_2}\right) = -|U_g| \frac{R_1}{R_1 + R_2} = U_{R_1}$$

Primer:



$|U_{g1}|, |U_{g2}|, R_1, R_2, R_3; \quad R_{\check{z}} \ll R_{1/2/3}$

$$\boxed{\begin{aligned} C_1&: |U_{g1}| - |U_{g2}| + I_2 R_2 - I_1 R_1 = 0 \\ C_2&: |U_{g1}| + I_3 R_3 - I_1 R_1 = 0 \end{aligned}}$$

$\times$ $\boxed{I_1 + I_2 + I_3 = 0}$ $\quad \} \cdot R_1 \quad \rightarrow \boxed{R_1 I_1 + R_1 I_2 + R_1 I_3 = 0}$

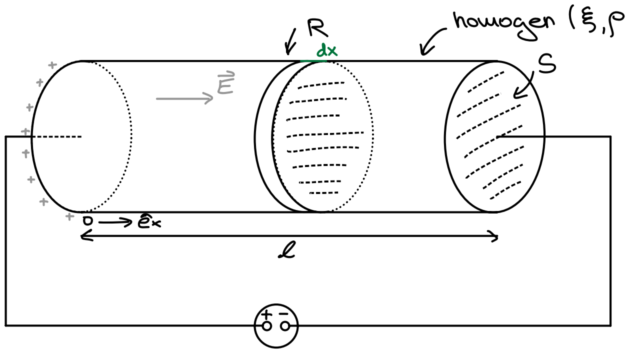
1. Kirchhoffov izrek

$$\rightarrow \begin{aligned} R_1 I_1 - R_2 I_2 &= |U_{g1}| - |U_{g2}| \\ R_1 I_1 - R_3 I_3 &= |U_{g1}| \\ R_1 I_1 + R_1 I_2 + R_1 I_3 &= 0 \end{aligned} \quad \rightarrow R = \begin{bmatrix} R_1 & -R_2 & 0 \\ R_1 & 0 & -R_3 \\ R_1 & R_1 & R_1 \end{bmatrix}; \quad \vec{I} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}; \quad \vec{U} = \begin{bmatrix} |U_{g1}| - |U_{g2}| \\ |U_{g1}| \\ 0 \end{bmatrix}$$

$R\vec{I} = \vec{U}; \quad \vec{I} = ?$

$\exists R^{-1} \Rightarrow \vec{I} = R^{-1}\vec{U}; \quad R^{-1} = ?$ (DN)

ELEKTRIČNO DELO IN ELEKTRIČNA MOČ



$\vec{E}$ homogeno $\Rightarrow \vec{F}_{el} = q_1 \vec{E}$

- $dV = S dx$
- $d\vec{l} = \vec{v} dt = d\vec{s}$
- $dq = \rho dV = \rho S dx$
- $d\vec{F}_{el} = dq \vec{E} = \rho S \vec{E} dx$
- $d^2 A_{el} = d\vec{F}_{el} \, d\vec{s} = \rho S \vec{E} \vec{v} dt dx = S \vec{j} \vec{E} dt dx$

Ohmov zakon: $\vec{j} = \frac{1}{\xi} \vec{E} \Rightarrow \vec{E} = \vec{j} \xi \qquad \Rightarrow d^2 A_{el} = \underbrace{S \xi \vec{j}}_{S\vec{j} = I_R} \cdot \vec{j} \, dt dx = I_R \cdot \frac{I_R}{S} \xi \, dt dx = \frac{\xi}{S} I_R^2 \, dx dt$

$$\Rightarrow \boxed{dA_{el} = dt \int_0^l \frac{\xi}{S} I_R^2 \, dx = dt \frac{\xi l}{S} I_R^2 = dt \, R I_R^2}$$

$$\Rightarrow \boxed{P_{el} = \frac{dA_{el}}{dt} = R I_R^2} \quad (>0) \quad (\vec{v} \parallel \vec{F}_{el})$$

$$U_R = \pm I_R R \rightarrow I_R = \pm \frac{U_R}{R} \qquad \Rightarrow \boxed{P_{el} = R \frac{U_R^2}{R^2} = \frac{U_R^2}{R}}$$

Primer: $I_R, U_R = \text{konst.}$

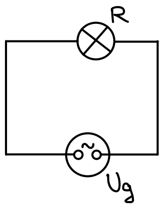
$$A_{el} = \int_0^t P_{el}(\tau)\,d\tau = R I_R^2 t = \frac{U_R^2}{R} t \qquad P_{el} \propto A_{el}$$

ENERGIJSKI ZAKON: $A_{el} + \underbrace{Q}_{\approx 0} = \Delta W_n \rightarrow A_{el} \approx \Delta W_n \sim m c_p \Delta T \rightarrow P_{el} \propto \Delta T$ (sevanje)

Poskus z izmeničnim tokom: $U_g = U_0 \sin(2\pi\nu t)$; $\frac{U_0}{\sqrt{2}} = 220\ V$

$\vec{E} = \vec{E}(t)$: $\oint \vec{E}\,d\vec{s} = 0$ predpostavimo, da velja

a)



$U_g = U_0 \sin(\omega t)$; $\omega = 2\pi\nu = \frac{2\pi}{t_0}$

$U_R + U_g = 0 \Rightarrow U_R = -U_g = -U_0 \sin(\omega t)$

$I_R = \pm \frac{U_R}{R} = \mp \underbrace{\frac{U_0}{R}}_{= I_0} \sin(\omega t) = \mp I_0 \sin(\omega t)$

$$P_{el} = \frac{U_R^2}{R} = \frac{U_0^2}{R} \sin^2(\omega t)$$

$$A_{el} = \int_0^{t = n t_0} P_{el}(\tau)\,d\tau = \frac{U_0^2}{R} \int_0^{t = n t_0} \sin^2\left(\frac{2\pi}{t_0}\tau\right) d\tau = \frac{U_0^2}{R}\frac{t}{2} = \frac{U_0^2}{2R} t$$

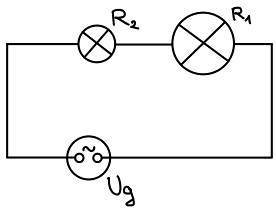
$A_{el} = \overline{P}_{el} \cdot t$;

$\Rightarrow \boxed{\overline{P}_{el} = \frac{1}{R}\frac{U_0^2}{2} = \frac{1}{R}\left(\frac{U_0}{\sqrt{2}}\right)^2} \qquad R = \left(\frac{U_0}{\sqrt{2}}\right)^2 \frac{1}{P_{el}}$

Podatek na žarnici: $\frac{U_0}{\sqrt{2}} = 230\ V \Rightarrow \overline{P}_{el} = 25\ W \rightarrow R \propto \frac{1}{P_{nom.}}$

nominalna moč

b)



$(R_1 < R_2)$

$R = R_1 + R_2$

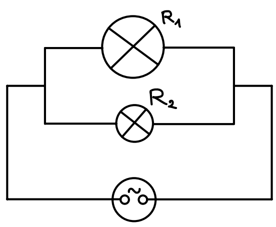
$I = \frac{U_g}{R} \Rightarrow U_{R_1} = I R_1 = \frac{U_g}{R} R_1 = U_0 \frac{R_1}{R} \sin(\omega t)$

$U_{R_2} = U_0 \frac{R_2}{R} \sin(\omega t)$

$\boxed{\overline{P}_1 = \frac{1}{2R_1}\left(\frac{U_0 R_1}{R}\right)^2 = \frac{1}{2R_1}\frac{U_0^2 R_1^2}{R^2} = \frac{U_0^2}{2R^2} R_1}$ in $\boxed{\overline{P}_2 = \frac{U_0^2}{2R^2} R_2}$

$R = R_1 + R_2 = R_2\left(1 + \frac{R_1}{R_2}\right) \rightarrow R^2 = R_2^2\left(1 + \frac{R_1}{R_2}\right)^2 \approx R_2^2 \Rightarrow \overline{P}_1 \approx \frac{U_0^2}{R_1 2}\frac{R_1^2}{R_2^2}$, $\overline{P}_2 \approx \frac{U_0^2}{2R_2}$

c)



$U_{R_1} = -U_g = -U_0 \sin(\omega t)$

$U_{R_2} = -U_0 \sin(\omega t)$

$\boxed{\overline{P}_1 = \frac{U_0^2}{2R_1} \qquad \overline{P}_2 = \frac{U_0^2}{2R_2}}$

sveti močneje

4. MAGNETNO POLJE

MAGNETNA SILA in GOSTOTA MAGNETNEGA POLJA

$\vec{F}_{el} = q\vec{E}_{q'}$; $\vec{F}_{el} \neq \vec{F}_{el}(\vec{v}_q)$

$S: \vec{v}_{q'} = 0$; $\vec{F} = \vec{F}_{el} = q\vec{E}_{q'} \neq \vec{F}(\vec{v}_q)$

$S': \vec{v}_0, \; \vec{v}_{q'} = -\vec{v}_0 \Rightarrow \vec{F}' = \vec{F}$

} Galilejeve transformacije

<u>Poskus:</u> dva vzporedna vodnika, oba nevtralna:
1. vodnik: naboj - izvori polja
2. vodnik: naboj - testni

(a) $\vec{v}_{q'} = \vec{0}$, $\vec{v}_2 = \vec{0}$, $\vec{F}_{el} = \vec{0}$

(b) $\vec{v}_2' = \vec{0}$, $\vec{v}_q \neq \vec{0}$, $\vec{F}_{el} = \vec{0}$

(c) $\vec{v}_q' \neq \vec{0}$; $\vec{v}_2' = \vec{0}$; $\vec{F} = \vec{0}$

(d) $\vec{v}_q' \neq \vec{0}$, $\vec{v}_2 \neq \vec{0}$; $\boxed{\vec{F} \neq \vec{0}}$ $\boxed{\vec{F} = \vec{F}(\vec{v}_2')}$

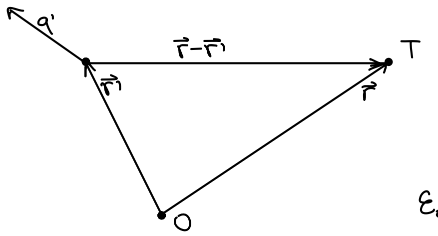
→ MAGNETNA SILA: $\vec{F}_m$; $\vec{F}_m \perp \vec{v}_q$

$$\boxed{\vec{F}_m = q\vec{v}_q \times \vec{B}}$$ (fizikalni zakon)

$N = \frac{Nm}{m} = \frac{J}{m} = \frac{VAs}{m}$

↖ GOSTOTA MAGNETNEGA POLJA $\left[\frac{VAs \cdot s}{m\,Asm} = \frac{Vs}{m^2} = T\right]$ ↖TESLA

$\vec{B}$ točkastega naboja q', ki se giblje s konstantno hitrostjo $\vec{v}_{q'}$
(Biot-Savart-ov zakon za točkast naboj)



$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0 q'}{4\pi} \vec{v}_{q'} \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}}$$

KONSTANTA MAGNETNEGA POLJA

$$\boxed{\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}}$$ $\left[\frac{s^2}{m^2}\right]$

$\varepsilon_0 = 8{,}9 \cdot 10^{-12} \frac{As}{Vm} \Rightarrow \boxed{c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \simeq 3 \cdot 10^8 \frac{m}{s}}$

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0 q'}{4\pi\varepsilon_0} \vec{v}_{q'} \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = \frac{1}{c_0^2} \vec{v}_{q'} \times \frac{q'}{4\pi\varepsilon_0} \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \frac{1}{c_0^2} \vec{v}_{q'} \times \vec{E}(\vec{r}) = \vec{\beta}_{q'} \times \frac{\vec{E}(\vec{r})}{c_0}}$$

$\vec{\beta}_{q'} = \frac{\vec{v}_{q'}}{c_0}$ brez dimenzije

$\vec{B}$ v primeru električnega toka (porazdeljenih nabojev):

$\vec{B}(\vec{r}) = \frac{\mu_0 q'}{4\pi} \vec{v}_{q'} \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$ $\rho(\vec{r}') \Rightarrow dq' = \rho(\vec{r}')\,dV$

$\rightarrow d\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \rho(\vec{r}) \cdot \vec{v}_{q'} \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} dV'$

$$\boxed{\vec{B}(\vec{r}) = \int_{\mathbb{R}^3} \frac{\mu_0}{4\pi} \vec{j}(\vec{r}') \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} dV'}$$ → načelo superpozicije

MAGNETNI (VEKTORSKI) POTENCIAL

- $\vec{E} \leftrightarrow \Phi$; $\vec{E} = -\nabla\Phi$

  ↑ skalarni potencial

- $\vec{B} \leftrightarrow \vec{A}$ ← vektorski potencial

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} \frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,dV'} \quad \left[\frac{Vs}{Am}\cdot\frac{A}{m^2}\cdot\frac{1}{m}\cdot m^3 = \frac{Vs}{m} = \frac{Vs}{m^2}m = Tm\right]$$

$$\Rightarrow \boxed{\vec{B} = \nabla\times\vec{A}}$$

neodvisno od x,y,z

$$\nabla\times\left(\vec{j}(\vec{r}\,')\frac{1}{|\vec{r}-\vec{r}\,'|}\right) = \underbrace{[\nabla\times\vec{j}(\vec{r}\,')]}_{0}\frac{1}{|\vec{r}-\vec{r}\,'|} - \vec{j}(\vec{r}\,')\times\nabla\frac{1}{|\vec{r}-\vec{r}\,'|}$$

$$\underline{\nabla\times\vec{A}} = \nabla\times\frac{\mu_0}{4\pi}\int_{\mathbb{R}^3}\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}dV' = \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3}\nabla\times\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}dV' = -\frac{\mu_0}{4\pi}\int_{\mathbb{R}^3}\vec{j}(\vec{r}\,')\times\nabla\frac{1}{|\vec{r}-\vec{r}\,'|}dV' =$$

$$\nabla\frac{1}{|\vec{r}-\vec{r}\,'|} = \frac{-(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

$$= \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3}\vec{j}(\vec{r}\,')\times\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}dV' = \underline{\vec{B}}$$

IZREK O MAGNETNEM PRETOKU

ploskev S: $\boxed{\int_S \vec{B}\,d\vec{S}}$ ← PRETOK MAGNETNEGA POLJA $[Vs]$

($\varepsilon_0\int_S \vec{E}\,d\vec{S}$ $[As]$ pretok električnega polja)

∂V … meja območja V (zaključena ploskev)

$\boxed{\oint_{\partial V}\vec{B}\,d\vec{S} = 0}$ ← IZREK O MAGNETNEM PRETOKU (integralska oblika)

↘ (diferencialna oblika)

$\vec{B} = \nabla\times\vec{A} \Rightarrow \boxed{\nabla\vec{B} = 0}$

$$\int_{\partial V}\vec{B}\,d\vec{S} = \int_V \nabla\vec{B}\,dV = \int \underbrace{\nabla(\nabla\times\vec{A})}_{0}dV = 0$$

Primer: $\nabla\vec{E} = \frac{\rho}{\varepsilon_0}$

$$\oint_{\partial V}\vec{E}\,d\vec{S} = \frac{q}{\varepsilon_0} \neq 0$$

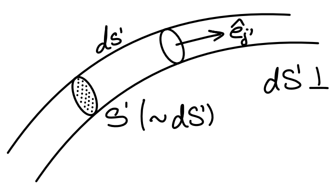
"silnice" $\vec{B}$ → gostotnice

BIOT-SAVARTOV ZAKON ZA TANKE ŽICE

$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3}\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}dV'}$; $\vec{j}(\vec{r}\,') = j(\vec{r}\,')\,\hat{e}_j$

$dV = dS'\,ds'$

$$= \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3}\frac{j(\vec{r}\,')\hat{e}_j}{|\vec{r}-\vec{r}\,'|}dS'ds' =$$



$dS' \perp ds' \Rightarrow j\,dS' = |dI|$

$$= \frac{\mu_0}{4\pi}\int_C\frac{\hat{e}_j\,ds'}{|\vec{r}-\vec{r}\,'|}\int_{S'} j(\vec{r}\,')\,dS' =$$

$$\boxed{= \frac{\mu_0|I|}{4\pi}\int_C\frac{d\vec{s}}{|\vec{r}-\vec{r}\,'|}} \;; \quad d\vec{s} = ds\cdot\hat{e}_j$$

$$\vec{B} = \nabla \times \vec{A} =$$

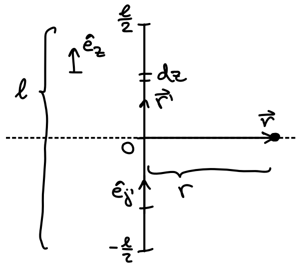
$$= \nabla \times \frac{\mu_0 |I|}{4\pi} \int_C \frac{d\vec{s}'}{|\vec{r}-\vec{r}'|} =$$

$$= \frac{\mu_0 |I|}{4\pi} \int_C \nabla \times \frac{\hat{e}_j'}{|\vec{r}-\vec{r}'|}\, ds' =$$

$$\nabla \times \frac{\hat{e}_j'}{|\vec{r}-\vec{r}'|} = (\nabla \times \hat{e}_j') \frac{1}{|\vec{r}-\vec{r}'|} + \nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) \times \hat{e}_j' = -\hat{e}_j' \times \nabla \frac{1}{|\vec{r}-\vec{r}'|} = = \hat{e}_j' \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\boxed{\vec{B} = \frac{\mu_0 |I|}{4\pi} \int_C d\vec{s}' \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$$ ← BIOT-SAVARTOV ZAKON v "originalni obliki"

Primer: $\vec{B}$ na simetrali dolgega ravnega vodnika s tokom:



Cilindrične koordinate: $\vec{r} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$

$\Downarrow$

$\vec{r} = \eta \hat{e}_\eta + \varphi \hat{e}_\varphi + z\hat{e}_z$, $\quad \hat{e}_\eta = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{bmatrix}$, $\quad \hat{e}_\varphi = \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix}$

$$1 = \hat{e}_\eta \cdot \hat{e}_\eta = \hat{e}_\varphi \cdot \hat{e}_\varphi = \hat{e}_z \cdot \hat{e}_z$$
$$\hat{e}_\eta \cdot \hat{e}_\varphi = \hat{e}_\varphi \cdot \hat{e}_z = \hat{e}_z \cdot \hat{e}_\eta = 0$$
$$\hat{e}_\varphi \times \hat{e}_z = \hat{e}_\eta$$
$$\hat{e}_\eta \times \hat{e}_\varphi = \hat{e}_z$$
$$\dot{\hat{e}}_\eta = \dot{\varphi}\, \hat{e}_\varphi \quad (\varphi = \sphericalangle\, \hat{e}_x, \hat{e}_\eta)$$

$$\Rightarrow |\vec{r}-\vec{r}'|^3 = \left(|\vec{r}-\vec{r}'|^2\right)^{\frac{3}{2}} = (r^2 + z'^2)^{\frac{3}{2}}$$

$$|\vec{r}-\vec{r}'|^2 = (\vec{r}-\vec{r}')(\vec{r}+\vec{r}') =$$
$$= (r\hat{e}_\eta - z'\hat{e}_z)(r\hat{e}_\eta + z'\hat{e}_z) =$$
$$= r^2 + z'^2$$

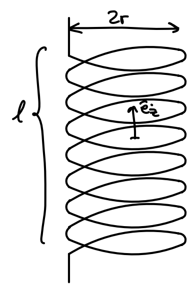
$$\Rightarrow \vec{r} = \eta \hat{e}_\eta + z\hat{e}_z = r\hat{e}_\eta$$
$$\vec{r}' = z'\hat{e}_z$$
$$d\vec{s} = dz'\hat{e}_z$$

$$\Rightarrow d\vec{s}' \times (\vec{r}-\vec{r}') = dz'\hat{e}_j' \times (\vec{r}-\vec{r}') =$$
$$= dz'\hat{e}_z \times (r\hat{e}_\eta - z'\hat{e}_z) =$$
$$= dz' r (\hat{e}_z \times \hat{e}_\eta) =$$
$$= dz' r\, \hat{e}_\varphi$$

$$\Rightarrow \boxed{\vec{B} =} \frac{\mu_0 |I|}{4\pi} \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{r\hat{e}_\varphi dz'}{(r^2+z'^2)^{\frac{3}{2}}} = \frac{\mu_0 |I|\, r\hat{e}_\varphi}{4\pi} \int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} \frac{dz'}{(r^2+z'^2)^{\frac{3}{2}}} = \frac{\mu_0 |I|\, r\hat{e}_\varphi}{4\pi} \cdot \frac{\ell}{r^2} \frac{1}{(r^2 + (\frac{\ell}{2})^2)^{\frac{1}{2}}} \simeq \frac{\mu_0 |I| r\hat{e}_\varphi \cdot 2}{4\pi \cdot r^2} = \boxed{\frac{\mu_0 |I|}{2\pi r}\hat{e}_\varphi}$$

$$\int \frac{dz'}{(r^2+z'^2)^{\frac{3}{2}}} = \frac{1}{r^2} \frac{z'}{(r^2+z'^2)^{\frac{1}{2}}} + C \quad \text{(D.N)}$$

Primer: $\vec{B}$ v tuljavi s tokom



N … število navojev

ℓ … dolžina tuljave

I, $\hat{e}_j$

$2r \ll \ell$

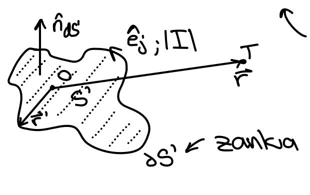
gosto navita: $\frac{N}{\ell}$

$$\vec{B} = \mu_0 \frac{N}{\ell} |I| \hat{e}_B \quad ; \quad \hat{e}_B = \begin{cases} +\hat{e}_z \,; & \hat{e}_j = \hat{e}_\varphi \\ -\hat{e}_z \,; & \hat{e}_j = -\hat{e}_\varphi \end{cases}$$

$$\boxed{\vec{B} = \mu_0 \frac{N}{\ell} |I| \hat{e}_z (\hat{e}_j \cdot \hat{e}_\varphi)}$$ ← velja znotraj tuljave

$\vec{B} \approx 0$ izven tuljave (zanemarljivo majhno v primerjavi s tistim skozi tuljavo)

Primer: $\vec{B}$ v okolici drobne zanke s tokom



$|\vec{r}| \gg |\vec{r}'|$ $\forall \vec{r}'$

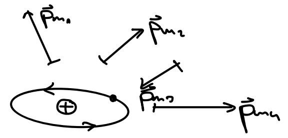
MAGNETNI DIPOLNI MOMENT

$$\boxed{\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^3} \cdot \left[3\hat{e}_r(\vec{p}_m \cdot \hat{e}_r) - \vec{p}_m\right]}$$

$$\vec{p}_m = |I|\vec{S}' = |I| \int_{S'} \hat{n}_{dS'}\, dS' \qquad \hat{n}_{dS'} = \hat{n}_{S'} \rightarrow \vec{S}' = \hat{n}_{S'} S'$$

TRAJNI MAGNETI:

atom ~ tokovne zanke



$\frac{1}{n}\sum \vec p_{m_i} \neq \vec 0$

SNOV V ZUNANJEM MAGNETNEM POLJU

Primer: tuljava: $\vec B = \frac{\mu_0 N \cdot |I|}{\ell}\hat e_B$

snov (jedro): magnetizacija
$\Rightarrow$ dodaten pospešek k $\vec B$

$\vec B \rightarrow \vec B' = \mu \vec B$
μ ← PERMEABILNOST SNOVI

$\mu \gg 1$ (~ 3000)
↳ FEROMAGNETNE SNOVI

$\vec B = \frac{\mu \cdot \mu_0 N |I|}{\ell}\hat e_B$

INDUKTIVNOST (TULJAVE)

IDEJA: $I \rightarrow \vec B \rightarrow \Phi_B$ (premikajoči se naboji ustvarjajo $\vec B$)
$\Phi_B \propto I$

$\boxed{\Phi_B = \pm L \cdot I}$
L ← INDUKTIVNOST

Primer: (idealna) tuljava:

(a) $\vec B = \frac{\mu\mu_0 N}{\ell}|I|\,\hat e_z\,(\hat e_j \cdot \hat e_\Phi)$

(b) $I = \vec j_e \cdot \vec S_{\check z} = \vec j_e \cdot \hat e_j \cdot \hat n_{\check z} \cdot S_{\check z} = \underbrace{j_e S_{\check z}}_{|I|} \cdot \hat e_j \hat n_{\check z} = |I|\,\underbrace{(\hat e_j \cdot \hat n_{\check z})}_{\pm 1}$

$(\hat e_j \hat n_{\check z}) I = |I|(\hat e_j \hat n_{\check z})(\hat e_j \hat n_{\check z})$
$\Rightarrow |I| = I(\hat e_j \hat n_{\check z})$

$\Rightarrow$ (a) in (b): $\vec B = \frac{\mu\mu_0 N I}{\ell}\hat e_z(\hat e_j \hat n_{\check z})(\hat e_j \hat e_\Phi)$

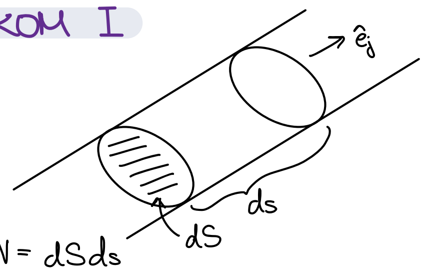
(c) $\Phi_B = N \vec B \vec S_1 = \underbrace{N\frac{\mu\mu_0 N S_1 I}{\ell}}(\hat e_j \hat n_{\check z})(\hat e_j \hat e_\Phi)(\hat e_z \hat n_1)$

$\boxed{L = \frac{\mu\mu_0 N^2 S_1}{\ell}}$ ← INDUKTIVNOST (IDEALNE) TULJAVE

$\left[\frac{\mathrm{Vs\,m^2}}{\mathrm{A\,m\,m}} = \mathrm{H}\right]$
↖ Henry

$\Phi_B = L I(\hat e_j \hat n_{\check z})(\hat e_j \hat e_\Phi)(\hat e_z \hat n_1)$

MAGNETNA SILA NA VODNIKE S TOKOM I



$\vec B, q, \vec v$ $\boxed{\vec F_m = q\vec v \times \vec B}$

$d^3q = \rho dV$; $\rho = nq_1$

$d^3F = d^3q\vec v \times \vec B = \rho\vec v \times \vec B dV = \vec j_e \times \vec B dV$

$dV = dS\,ds$

tanka žica:
$\vec B \approx$ konst. na celotnem S

$\Rightarrow \vec F = \int_V \vec j_e \times \vec B\,dV = \int_C\left[\int_S(j_e \hat e_j \times B)dS\right]ds = \int_C\left[\underbrace{\int_S j\,dS}_{|I|}\right](\hat e_j \times \vec B)ds =$

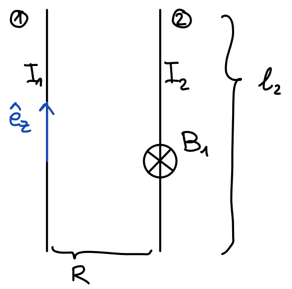
$= |I|\int_C \hat e_j \times B\,ds = |I|\int_C d\vec s \times \vec B$; $d\vec s = \hat e_j ds$

Primer: $\vec{B}$ = konst., C = ravna črta

$$\rightarrow \vec{F} = |I|\int_C d\vec{s}\times\vec{B} = |I|\left[\int_C d\vec{s}\right]\times\vec{B} = |I|\,\vec{\ell}\times\vec{B}\ ; \quad |\vec{\ell}| = \ell \text{ ... dolžina vodnika v } \vec{B}$$

$$\Rightarrow \underline{\vec{F} = |I|\,\ell\cdot\hat{e}_j\times\vec{B}}$$

Primer:



$R = 1\,m$

$\hat{e}_{j_1} = \hat{e}_{j_2} = \hat{e}_z$

$|I_1| = |I_2| = |I|$

$\ell_2 = 1\ m$

(a) $\vec{B}_1$ na mestu 2. vodnika

$$\vec{B}_1 = \frac{\mu_0 |I|}{2\pi R}\hat{e}_\Phi(\hat{e}_{j_1}\cdot\hat{e}_z) = \frac{\mu_0|I|}{2\pi R}\hat{e}_\Phi$$

(b) $\vec{\ell}_2 = \ell_2\cdot\hat{e}_{j_2} = \ell_2\cdot\hat{e}_z$

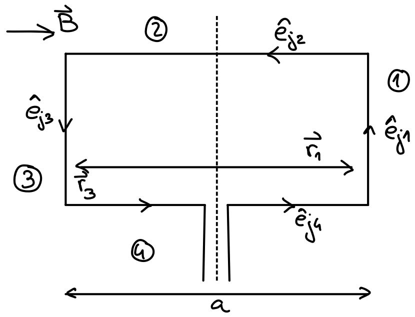
$$\rightarrow \vec{F} = |I_2|\,\vec{\ell}_2\times\vec{B}_1 = |I|\,\ell_2\,\hat{e}_z\times\frac{\mu_0|I|}{2\pi R}\hat{e}_\Phi = \frac{\mu_0|I|^2}{2\pi R}\ell\,\underbrace{\hat{e}_z\times\hat{e}_\Phi}_{-\hat{e}_n} = \frac{\mu_0|I|^2}{2\pi}\frac{\ell}{R}(-\hat{e}_n)$$

↑ sila privlačna

- $|I| = 1\,A \Rightarrow F = \frac{\mu_0|I|^2}{2\pi} = \frac{4\pi\cdot10^{-7}\,Vs\,A^2}{2\pi\ \ Am} = 2\cdot10^{-7}\,\frac{VAs}{m}$ N

NAVOR NA $\vec{p}_m$ (magnetni dipol)

zanka s tokom v homogenem magnetnem polju:



$\vec{B} = B\hat{e}_x$

$\vec{\ell}_1 = b\hat{e}_y$

$\vec{\ell}_2 = a(-\hat{e}_x)$

$\vec{\ell}_3 = b(-\hat{e}_y)$

$\vec{\ell}_4 = a\hat{e}_x$

$$\left.\begin{array}{l} \vec{F}_1 = |I|\,\vec{\ell}_1\times\vec{B} = |I|bB\hat{e}_y\times\hat{e}_x = |I|bB(-\hat{e}_z) \\ \vec{F}_2 = 0 \\ \vec{F}_3 = -\vec{F}_1 = |I|bB\hat{e}_z \\ \vec{F}_4 = 0 \end{array}\right\} \Rightarrow \sum F_i = 0 \qquad \vec{F}_1 = -\vec{F}_3 \text{ ... dvojica sil}$$

$\vec{r}_1 = \frac{a}{2}\hat{e}_x$ in $\vec{r}_2 = \frac{a}{2}(-\hat{e}_x)$

$$\left.\begin{array}{l} \vec{M}_1 = \vec{r}_1\times\vec{F}_1 = (-\frac{a}{2})|I|bB\,\hat{e}_x\times\hat{e}_z = \frac{1}{2}ab\,|I|B\cdot\hat{e}_y \\ \vec{M}_3 = \vec{r}_3\times\vec{F}_3 = \frac{1}{2}ab\,|I|B\cdot\hat{e}_y \end{array}\right\} \Rightarrow \sum\vec{M}_i = \vec{M}_1 + \vec{M}_3 = ab|I|B\hat{e}_y \neq 0$$

$ab = S \Rightarrow \vec{M} = S\cdot|I|\cdot B\,\hat{e}_y$

Spomnimo se: $\vec{p}_m = S\cdot|I|\,\hat{n}_s$; $\hat{n}_s \perp$ ravnina zanke $\rightarrow \hat{n}_s = \hat{e}_z$

$$\boxed{\vec{p}_m\times\vec{B} = S|I|\hat{e}_z\times B\hat{e}_x = S|I|B\,\hat{e}_z\times\hat{e}_x = S|I|B\cdot\hat{e}_y = \vec{M}}$$

ENERGIJA $\vec{p}_m$ V $\vec{B}$

? ? ?

MAGNETNA NAPETOST

Elektrostatika: $\vec{\nabla}\cdot\vec{E} = \frac{\rho_e}{\varepsilon_0}$ $\oint_{\partial V} \vec{E}\,d\vec{s}$

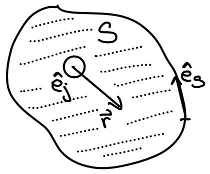
$\vec{\nabla}\times\vec{E} = 0 \quad \left(\oint_{\partial S} \vec{E}\,d\vec{s} = 0\right)$

Magnetostatika: $\vec{\nabla}\cdot\vec{B} = 0 \quad \left(\oint_{\partial V} \vec{B}\,d\vec{s} = 0\right)$

$\vec{\nabla}\times\vec{B} = ? \quad \left(\oint_{\partial S} \vec{B}\,d\vec{s} = ?\right)$

DEFINICIJA: $\boxed{\oint_C \vec{B}\,d\vec{s} \equiv \text{MAGNETNA NAPETOST PO } C}$

Primer:



$$\oint_{\partial S} \vec{B}\,d\vec{s} = \int_{t_1}^{t_2} \vec{B}\dot{\vec{r}}\,dt$$

Cilindrični koordinatni sistem: $z=0 \Rightarrow \vec{r} = r\hat{e}_r \Rightarrow \dot{\vec{r}} = \dot{r}\hat{e}_r + r\dot{\hat{e}}_r = \dot{r}\hat{e}_r + r\dot{\Phi}\hat{e}_\Phi$

$$\Rightarrow \vec{B}\dot{\vec{r}} = \frac{\mu_0|I|}{2\pi r}\hat{e}_\Phi(\dot{r}\hat{e}_r + r\dot{\Phi}\hat{e}_\Phi) = \frac{\mu_0|I|}{2\pi}\dot{\Phi}$$

$$\Rightarrow \oint_{\partial S} \vec{B}\,d\vec{s} = \frac{\mu_0|I|}{2\pi}\int_{t_1}^{t_2}\dot{\Phi}\,dt = \frac{\mu_0|I|}{2\pi}\underbrace{(\Phi(t_2)-\Phi(t_1))}_{2\pi} = \mu_0|I|$$

2π če zanka objame vodnik, sicer 0 (n-krat okoli $\rightarrow$ $\times n$)

$$\boxed{\oint_{\partial S}\vec{B}\,d\vec{s} = \mu_0|I|}$$

zaobjeti tok $\rightarrow I = \int_S \vec{j}_e\,d\vec{s}$

$$\Rightarrow \boxed{\oint_{\partial S}\vec{B}\,d\vec{s} = \mu_0\int_S \vec{j}_e\,d\vec{s}}$$

$$\oint \vec{B}\,d\vec{s} = \int_S (\vec{\nabla}\times B)\,d\vec{s} = \mu_0\int \vec{j}_e\,d\vec{s}\,;\ \forall S \quad \Rightarrow \quad \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{j}_e}$$

$\leftarrow$ IZREK O MAGNETNI NAPETOSTI V DIFERENCIALNI OBLIKI

Predpostavke:

1) $\frac{\partial\rho_e}{\partial t} = 0 \Rightarrow \frac{\partial\rho}{\partial t} = -\vec{\nabla}\vec{j}_e = 0$

2) $\frac{\vec{j}(\vec{r}_i)}{|\vec{r}-\vec{r}_i|} \xrightarrow{\vec{r}\to\infty} 0$

3) $\nabla\int\ldots = \int D\ldots$

INVARIANCA $\vec{p}_m$ in GALILEJEVE TRANSFORMACIJE

(a) $\vec{F} = m\vec{a}\,;$ v vseh inercialnih opazovalnih sistemih

(b) Galilejeve transformacije: $S, S', \vec{v}_0, \vec{\beta}_0 := \frac{\vec{v}_0}{c_0}\,;\ c_0 := \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

$t=0:\ S=S',\ t=t' \quad (c_0t = c_0t')$

$\vec{r}' = \vec{r} - \vec{v}_0t = \vec{r} - \vec{\beta}_0c_0t$

(c) Galilejeve transformacije: $\vec{v}$ in $\vec{a}$: $\vec{v} = \frac{d\vec{r}}{dt} \Rightarrow \vec{v}' = \frac{d\vec{r}'}{dt'} = \frac{d\vec{r}'}{dt} = \frac{d\vec{r}}{dt} - \frac{d}{dt}(\vec{v}_0t) = \vec{v} - \vec{v}_0$

$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \vec{a}' = \frac{d\vec{v}'}{dt'} = \frac{d\vec{v}'}{dt} = \frac{d\vec{v}}{dt} - \frac{d\vec{v}_0}{dt} = \vec{a}$ $\leftarrow$ Galilejeva invarianca

(d) Galilejeve transformacije: m in q: $\underbrace{m'=m}$ in $\underbrace{q'=q}$ (eksperimentalno dejstvo)

$$\Rightarrow \vec{F} = m\vec{a} = m'\vec{a}' = \vec{F}' \leftarrow \text{Galilejeva invarianca}$$

(e) magnetna sila:

primer: $S: q, \vec{v}_q, \vec{B} \rightarrow \vec{F}_m = q\vec{v}_q \times \vec{B} = qc_0 \frac{\vec{v}_q}{c_0} \times \vec{B} = qc_0 \vec{\beta}_q \times \vec{B}$

$S': \vec{v}_0 = \vec{v}_q \Rightarrow \vec{v}_{q'} = \vec{v}_q - \vec{v}_0 = 0 \rightarrow \vec{F}_{m'} = q\vec{v}_{q'} \times \vec{B}' = 0$

$\Big\} \Rightarrow \vec{F}_{m'} \neq \vec{F}_m$

Rešitev:

(a) $\vec{F}_L := \vec{F}_m + \vec{F}_e = q\vec{E} + q\vec{v}_q \times \vec{B} = qc_0\left(\frac{\vec{E}}{c_0} + \vec{\beta}_q \times \vec{B}\right)$

(b) $S: \vec{E} = 0 \Rightarrow \vec{F}_L = qc_0 \vec{\beta}_q \times \vec{B}$

$S': \vec{E}', \vec{B}' \rightarrow \vec{F}_L' = qc_0\left(\frac{\vec{E}'}{c_0} + \vec{\beta}_q' \times \vec{B}'\right) \overset{\vec{v}_q'=0}{=} qc_0\left(\frac{\vec{E}'}{c_0}\right)$

$\vec{\beta}_q = \frac{\vec{v}_q}{c_0} = \frac{\vec{v}_0}{c_0} = \vec{\beta}_0$

$\vec{\beta}_q' = \frac{\vec{v}_q'}{c_0} = 0$

$$\rightarrow \vec{F}_L = \vec{F}_L' \Rightarrow \frac{\vec{E}'}{c_0} = \vec{\beta}_q \times \vec{B} = \frac{\vec{E}}{c_0} + \vec{\beta}_0 \times \vec{B}$$

BIOT-SAVARTOV ZAKON ZA TOČKASTE NABOJE: q izvor polja (ne testni naboj); $\vec{r}_q$

$S: \vec{v}_q = 0, \ \vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}_q}{|\vec{r}-\vec{r}_q|^3}, \quad \vec{B}(\vec{r}) = 0$

$S': \vec{v}_0, \ \vec{v}_q' = \vec{v}_q - \vec{v}_0 = -\vec{v}_0 \quad (\vec{\beta}_q' = -\vec{\beta}_0); \quad \vec{B}(\vec{r}') = \vec{v}_q' \times \frac{\mu\ell}{4\pi} \frac{\vec{r}-\vec{r}_q}{|\vec{r}-\vec{r}_q|^3} = \frac{\vec{v}_0}{c_0} \times \frac{1}{c_0} \frac{q}{4\pi\varepsilon_0} \frac{\vec{r}-\vec{r}_q}{|\vec{r}-\vec{r}_q|^3} =$

$= -\vec{\beta}_0 \times \frac{1}{c_0} \vec{E}(\vec{r})$

$$\rightarrow \boxed{\vec{B}' = -\vec{\beta}_0 \times \frac{\vec{E}(\vec{r})}{c_0} = \vec{B} - \vec{\beta}_0 \times \frac{\vec{E}}{c_0}}$$

Splošno:

$$\vec{F}_L = qc_0\left(\frac{\vec{E}}{c_0} + \vec{\beta}_q \times \vec{B}\right); \quad \vec{\beta}_q = \frac{\vec{v}_q}{c_0}$$

$$\vec{F}_L' = q'c_0\left(\frac{\vec{E}'}{c_0} + \vec{\beta}_q' \times \vec{B}'\right) =$$

$$= qc_0\left[\frac{\vec{E}}{c_0} + \vec{\beta}_0 \times \vec{B} + (\vec{\beta}_q - \vec{\beta}_0) \times \left(\vec{B} - \vec{\beta}_0 \times \frac{\vec{E}}{c_0}\right)\right] =$$

$$= qc_0\left[\frac{\vec{E}}{c_0} + \vec{\beta}_0 \times \vec{B} + \vec{\beta}_q \times \vec{B} - \vec{\beta}_0 \times \vec{B} + (\vec{\beta}_q - \vec{\beta}_0) \times \left(\vec{\beta}_0 \times \frac{\vec{E}}{c_0}\right)\right] =$$

$$= qc_0\left[\frac{\vec{E}}{c_0} + \vec{\beta}_q \times \vec{B} + (\vec{\beta}_q - \vec{\beta}_0) \times \left(\vec{\beta}_0 \times \frac{\vec{E}}{c_0}\right)\right] =$$

$$= \vec{F}_L + \Delta\vec{F}_L$$

$$\Delta\vec{F}_L = qc_0\left(\vec{\beta}_q - \vec{\beta}_0\right) \times \left(\vec{\beta}_0 \times \frac{\vec{E}}{c_0}\right) \neq 0$$

v splošnem

Velja: $\Delta\vec{F}_L = (\vec{\beta}_q - \vec{\beta}_0) \times (\vec{\beta}_0 \times \vec{F}_L)$

$$\Rightarrow \vec{F}_L' \neq \vec{F}_L$$

Imamo $f(x,y)$ in $f(x_0,y_0)$. $\Rightarrow f(x,y) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0) + R_2$

$$\Rightarrow \boxed{f(x,y) \approx f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0)}$$

$$\vec{F}_L' = \vec{F}_L'(\vec{\beta}_q, \vec{\beta}_0) = \begin{bmatrix} F'_{L,x}(\vec{\beta}_q, \vec{\beta}_0) \\ F'_{L,y}(\vec{\beta}_q, \vec{\beta}_0) \\ F'_{L,z}(\vec{\beta}_q, \vec{\beta}_0) \end{bmatrix}$$

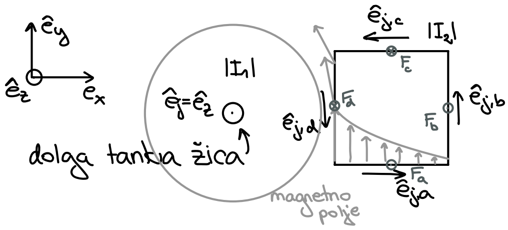
$$F'_{L,x}(\vec{\beta}_q, \vec{\beta}_0) \approx F'_{L,x}(0,0) + \frac{\partial F'_{L,x}}{\partial \beta_{q,x}}(0,0)\beta_{q,x} + \frac{\partial F'_{L,x}}{\partial \beta_{q,y}}(0,0)\beta_{q,y} + \frac{\partial F'_{L,x}}{\partial \beta_{q,z}}(0,0)\beta_{q,z} +$$

$$+ \frac{\partial F'_{L,x}}{\partial \beta_{0,x}}(0,0)\beta_{0,x} + \frac{\partial F'_{L,x}}{\partial \beta_{0,y}}(0,0)\beta_{0,y} + \frac{\partial F'_{L,x}}{\partial \beta_{0,z}}(0,0)\beta_{0,z} = F_{L,x}(\vec{\beta}_q, \vec{\beta}_0)$$

Podobno: $F'_{L,y}(\vec{\beta}_q,\vec{\beta}_0) \approx F_{L,y}(\vec{\beta}_q,\vec{\beta}_0)$ in $F'_{L,z}(\vec{\beta}_q,\vec{\beta}_0) \approx F_{L,z}(\vec{\beta}_q,\vec{\beta}_0)$

$\boxed{F_L: \text{ invariantna na Galilejevo transformacijo v linearnem približku}; \ \beta_q,\beta_0 \ll 1}$

MAGNETNA SILA in 3. NEWTONOV ZAKON



$\vec{F}_{1\to 2} \propto \hat{e}_z$
$\vec{F}_{2\to 1} \perp \hat{e}_z$ } različne smeri
$\Rightarrow$ ne velja 3. Newtonov zakon

REORGANIZACIJA POGLAVJA

Biot-Savartov zakon: $\vec{B}(\vec{r}) = \int_{\mathbb{R}^3} \frac{\mu_0}{4\pi} \vec{j}(\vec{r}\,') \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} dV'$

$\Rightarrow$ izrek: $\vec{\nabla}\cdot\vec{B} = 0$

$\Rightarrow$ izrek: $\vec{\nabla}\times\vec{B} = \mu_0 \vec{j} \quad (\vec{\nabla}\vec{j} = 0, \ \frac{\partial\rho}{\partial t} = 0)$

Obrnemo sklepanje:

a) zakon (o magnetnem pretoku): $\vec{\nabla}\cdot\vec{B} = 0$

b) zakon: $\vec{\nabla}\times\vec{B} = \mu_0\vec{j}$

$\Rightarrow$ Biot-Savartov izrek: $\vec{B}(\vec{r}) = \int_{\mathbb{R}^3} \frac{\mu_0}{4\pi} \vec{j}(\vec{r}\,') \times \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} dV'$

Dokaz:

Helmholzov izrek: $\vec{B}(\vec{r}) = -\vec{\nabla}\left[\frac{1}{4\pi}\int_{\mathbb{R}^3} \frac{\vec{\nabla}'\vec{B}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dV'\right] + \vec{\nabla}\times\left[\frac{1}{4\pi}\int_{\mathbb{R}^3} \frac{\vec{\nabla}'\times\vec{B}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dV'\right] =$

$$= \vec{\nabla}\times\left[\frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} \frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dV'\right] = \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} \vec{\nabla}\times\frac{\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dV' =$$

$$= \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} \frac{\vec{\nabla}\times\vec{j}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} - \vec{j}(\vec{r}\,')\times\nabla\frac{1}{|\vec{r}-\vec{r}\,'|} dV' =$$

$$= \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} \vec{j}(\vec{r}\,')\times\frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} dV' \quad \square$$

POVZETEK II. – IV.

II: $\vec{\nabla}\times\vec{E} = 0$
$\vec{\nabla}\vec{E} = \frac{\rho_e}{\varepsilon_0}$
$\vec{F}_q = q\vec{E}$

III: $\vec{E} = \xi\vec{j}$
$\frac{\partial\rho}{\partial t} = -\vec{\nabla}\vec{j}$

IV: $\vec{\nabla}\vec{B} = 0$
$\vec{\nabla}\times\vec{B} = \mu_0\vec{j}$
$\vec{F}_L = q(\vec{E} + \vec{v}_q\times\vec{B}) = qc_0\left(\frac{\vec{E}}{c_0} + \vec{\beta}_q\times\vec{B}\right)$
$\frac{\vec{E}'}{c_0} \approx \frac{\vec{E}}{c_0} + \vec{\beta}_0\times\vec{B}$
$\vec{B}' \approx \vec{B} - \vec{\beta}_0\times\frac{\vec{E}}{c_0}$

5. ELEKTRODINAMIKA

(1) $\boxed{\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0}}$ ... zakon o električnem pretoku ← 1. MAXWELLOVA ENAČBA

(2) $\vec{\nabla}\times\vec{E} = 0$ ... zakon o električni napetosti

(3) $\boxed{\vec{\nabla}\cdot\vec{B} = 0}$ ... zakon o magnetnem pretoku ← 2. MAXWELLOVA ENAČBA

(4) $\vec{\nabla}\times\vec{B} = \mu_0\vec{j}$ ... zakon o magnetni napetosti

ZAKON O MAGNETNI NAPETOSTI

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} \quad /\vec{\nabla}$$

$$\rightarrow \underbrace{\vec{\nabla}(\vec{\nabla}\times\vec{B})}_{0} = \mu_0\vec{\nabla}\vec{j} \Rightarrow \boxed{\vec{\nabla}\vec{j} = 0}$$

Po drugi strani: $\vec{\nabla}\vec{j} = -\frac{\partial\rho}{\partial t} \neq 0$

$\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0\vec{j}$ ne velja v splošnem

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \vec{G} \quad /\vec{\nabla}$$

$$\rightarrow 0 = \mu_0\vec{\nabla}\vec{j} + \vec{\nabla}\vec{G} \qquad \Rightarrow \boxed{\vec{\nabla}\vec{G} = -\mu_0\vec{\nabla}\vec{j} = \mu_0\frac{\partial\rho}{\partial t}}$$

$$\frac{\partial\rho}{\partial t} = \varepsilon_0\frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{E}) = \varepsilon_0\vec{\nabla}\left(\frac{\partial\vec{E}}{\partial t}\right) =$$

$$\Rightarrow \boxed{\mu_0\frac{\partial\rho}{\partial t} = \mu_0\varepsilon_0\vec{\nabla}\left(\frac{\partial\vec{E}}{\partial t}\right) = \frac{1}{c_0^2}\vec{\nabla}\left(\frac{\partial\vec{E}}{\partial t}\right)}$$

$$\Rightarrow \vec{\nabla}\vec{G} = \frac{1}{c_0^2}\vec{\nabla}\left(\frac{\partial\vec{E}}{\partial t}\right)$$

$$\Rightarrow \boxed{\vec{G} = \frac{1}{c_0^2}\frac{\partial\vec{E}}{\partial t}}$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \frac{1}{c_0^2}\frac{\partial\vec{E}}{\partial t}}$$ ← 3. MAXWELLOVA ENAČBA

V integralni obliki: $\boxed{\oint_{\partial S}\vec{B}\,d\vec{s} = \int(\vec{\nabla}\times\vec{B})\,d\vec{S} = \mu_0\int_S\vec{j}\,d\vec{S} + \frac{1}{c_0^2}\int_S\frac{\partial\vec{E}}{\partial t}\,d\vec{S}}$

V primeru, da $S \neq S(t)$: $\int\frac{\partial\vec{E}}{\partial t}\,d\vec{S} = \frac{d}{dt}\int\vec{E}\,d\vec{S} = \frac{1}{\varepsilon_0}\frac{d}{dt}\Phi_E$

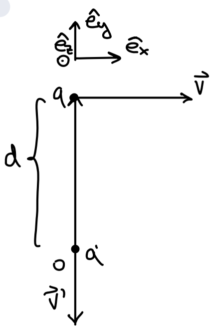
$$\vec{\nabla}(\vec{\nabla}\times\vec{B}) = 0 = \mu_0\vec{\nabla}\vec{j} + \frac{1}{c_0^2}\vec{\nabla}\left(\frac{\partial\vec{E}}{\partial t}\right) = \mu_0\vec{\nabla}\vec{j} + \frac{1}{c_0^2}\frac{\partial}{\partial t}(\vec{\nabla}\vec{E}) = \mu_0\vec{\nabla}\vec{j} + \frac{1}{c_0^2}\frac{1}{\varepsilon_0}\frac{\partial}{\partial t}(\rho) = \mu_0\left(\vec{\nabla}\vec{j} + \frac{\partial}{\partial t}\rho\right) = 0$$

$$\Rightarrow \vec{\nabla}\vec{j} + \frac{\partial\rho}{\partial t} = 0$$

$$\Rightarrow \frac{\partial\rho}{\partial t} = -\vec{\nabla}\vec{j}$$

(III/12) F_L in 3. NEWTONOV ZAKON

(nekaj od prej /dodatek/)



Primer: q': $\vec r' = 0$, $\vec v' = v'(-\hat e_y)$ q: $\vec r = d\hat e_y$, $\vec v = v\,\hat e_x$

$$\vec r - \vec r' = d\hat e_y$$
$$\vec r' - \vec r = d(-\hat e_y)$$
$$|\vec r - \vec r'| = d$$

$$\vec E_{q'}(\vec r) = \frac{q'}{4\pi\varepsilon_0 d^2}\hat e_y$$
$$\vec E_q(\vec r') = \frac{q}{4\pi\varepsilon_0 d^2}(-\hat e_y)$$

$$\vec B_{q'}(\vec r) = \frac{1}{c_0^2}\vec v' \times \vec E_{q'}(\vec r) = \frac{q'v'}{4\pi\varepsilon_0 d^2}(-\hat e_y)\times \hat e_y = 0$$
$$\vec B_q(\vec r') = \frac{1}{c_0^2}\vec v \times \vec E_q(\vec r') = \frac{qv}{c_0^2 4\pi\varepsilon_0 d^2}\hat e_x \times (-\hat e_y) = \frac{qv}{c_0^2 4\pi\varepsilon_0 d^2}(-\hat e_z)$$

$$\vec F_e(q'\to q) = q\vec E_{q'} = \frac{qq'}{4\pi\varepsilon_0 d^2}\hat e_y$$
$$\vec F_e(q\to q') = q'\vec E_q = \frac{q'q}{4\pi\varepsilon_0 d^2}(-\hat e_y) = -\vec F_e(q'\to q)$$ ← 3. Newtonov zakon velja

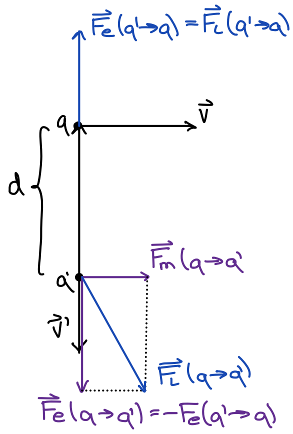
$$\vec F_m(q'\to q) = q\vec v \times \vec B_{q'} = 0$$
$$\vec F_m(q\to q') = q'\vec v' \times \vec B_q = \frac{q'q}{4\pi\varepsilon_0 d^2}\frac{v'v}{c_0^2}(-\hat e_y)\times(-\hat e_z) = \frac{q'q}{4\pi\varepsilon_0 d^2}\beta\beta'\,\hat e_x$$ ← 3. Newtonov zakon ne velja

$$\vec F_L(q'\to q) \equiv \vec F_e(q'\to q) + \vec F_m(q'\to q)$$
$$\vec F_L(q\to q') \equiv \vec F_e(q\to q') + \vec F_m(q\to q')$$
$$\Rightarrow \vec F_L(q'\to q) \neq \vec F_L(q\to q')$$
↑ 3. Newtonov zakon ne velja.

TEŽAVA: (sunek sile)
$$\int \vec F_L(q'\to q)\,dt = \Delta\vec G_q = \vec G_{qk} - \vec G_{qz}$$
$$\int \vec F_L(q\to q')\,dt = \Delta\vec G_{q'} = \vec G_{q'k} - \vec G_{q'z} \neq -(\vec G_{qk} - \vec G_{qz})$$

$$\vec G_{qk} - \vec G_{qz} + \vec G_{q'k} - \vec G_{q'z} = \vec G_{qk} + \vec G_{q'k} - (\vec G_{qz} + \vec G_{q'z}) = \vec G_k - \vec G_z$$
$$\neq \vec G_{qk} - \vec G_{qz} - (\vec G_{qk} - \vec G_{qz}) = 0 \Rightarrow \boxed{\vec G_k \neq \vec G_z}$$

↑ skupna gibalna količina se ne ohranja



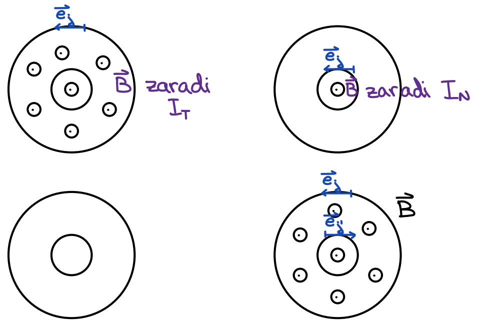
ZAKON O ELEKTRIČNI NAPETOSTI

$$\vec{\nabla}\times\vec{E}=0$$

Poskus: Faraday

Kvalitativno: vključimo/izključimo I
$\Rightarrow$ spremeni se $\vec{B}$; $\frac{\partial \vec{B}}{\partial t}\neq 0$
$\Rightarrow$ $\frac{\partial \vec{B}}{\partial t}\neq 0 \rightarrow$ se obda z $\vec{E}$ $\Rightarrow$ požene I_N

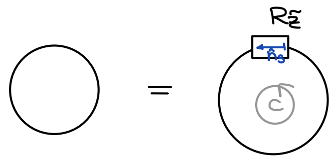
Tloris: (a) (b)



Lentzovo pravilo

Poskus v nasprotju z: $\vec{\nabla}\times\vec{E}=0$ in $\vec{E}=\mathcal{E}_j$

- notranja zanka:



$\Rightarrow U=-I\cdot R_\Xi\neq 0$

$\vec{\nabla}\times\vec{E}=0$

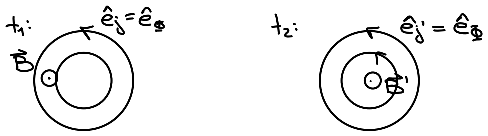
$\Rightarrow \int(\vec{\nabla}\times\vec{E})\,d\vec{S}=0$

$\Rightarrow \oint_{\partial S}\vec{E}\,d\vec{s}=\int(\vec{\nabla}\times\vec{E})\,d\vec{S}=0=-U$

$\Rightarrow U=0$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}} \leftarrow \text{4. MAXWELOVA ENAČBA}$$

$$\int_S(\vec{\nabla}\times\vec{E})\,d\vec{S}=\boxed{-\int\frac{\partial\vec{B}}{\partial t}\,d\vec{S}=\oint_{\partial S}\vec{E}\,d\vec{s}} \leftarrow \text{integralna oblika}$$



a) $\vec{B}(t_1)=B\hat{e}_z$ $\quad B(t_2)=0$

$\Rightarrow \Delta\vec{B}=\vec{B}(t_2)-\vec{B}(t_1)=B(-\hat{e}_z)$

$\Rightarrow \frac{\partial\vec{B}}{\partial t}\propto(-\hat{e}_z) \Rightarrow \boxed{-\frac{\partial\vec{B}}{\partial t}\propto\hat{e}_z} \Rightarrow \boxed{\vec{E}\propto\hat{e}_\Phi}$

$\hat{e}_\Phi=\sin\Phi\,\hat{e}_x+\cos\Phi\,\hat{e}_y$

$\sin\Phi=\frac{y}{r}$

$\cos\Phi=\frac{x}{r}$

$r=\sqrt{x^2+y^2}$

$\rightarrow \frac{\partial}{\partial x}(\sin\Phi)=\frac{xy}{r^2}$ $\qquad \frac{\partial}{\partial x}(\cos\Phi)=\frac{1}{r}-\frac{x^2}{r^2}$

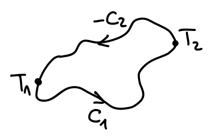
$\frac{\partial}{\partial y}(\sin\Phi)=\frac{1}{r}-\frac{y^2}{r^2}$ $\qquad \frac{\partial}{\partial y}(\cos\Phi)=\frac{xy}{r^2}$

$\frac{\partial}{\partial z}(\sin\Phi)=0$ $\qquad \frac{\partial}{\partial z}(\cos\Phi)=0$

$$\vec{\nabla}\times\vec{E}\propto\vec{\nabla}\times\hat{e}_\Phi=\left(\frac{\partial}{\partial y}0-\frac{\partial}{\partial z}\sin\Phi\right)\hat{e}_x+\left(-\frac{\partial}{\partial z}(\sin\Phi)-\frac{\partial}{\partial x}0\right)\hat{e}_y+\left(\frac{\partial}{\partial x}(\cos\Phi)+\frac{\partial}{\partial y}(\sin\Phi)\right)\hat{e}_z=$$

$$=\left(\frac{1}{r}-\frac{x^2}{r^2}+\frac{1}{r}-\frac{y^2}{r^2}\right)\hat{e}_z=\left(\frac{2}{r}-\frac{(x^2+y^2)}{r^3}\right)\hat{e}_z=\left(\frac{2}{r}-\frac{1}{r}\right)\hat{e}_z=\frac{1}{r}\hat{e}_z$$

Komentarji: a) $\vec{\nabla}\times\vec{E}\neq 0 \Rightarrow -\oint_{\partial S}\vec{E}\,d\vec{s} = \int_S \frac{\partial\vec{B}}{\partial t}\,d\vec{S} \neq 0$ (V nasprotju z 2. Kirchoffovim izrekom)

b) 

$$\partial S = C_1 + (-C_2)$$

$$\oint_{\partial S}\vec{E}\,d\vec{s} = \int_{C_1}\vec{E}\,d\vec{s} + \int_{-C_2}\vec{E}\,d\vec{s} = \int_{C_1}\vec{E}\,d\vec{s} - \int_{C_2}\vec{E}\,d\vec{s} \neq 0$$

$$\Rightarrow \int_{C_1}\vec{E}\,d\vec{s} \neq \int_{C_2}\vec{E}\,d\vec{s}$$

c) $-U(T_1\to T_2; C_1) \neq -U(T_1\to T_2; C_2)$

$\boxed{U(T_1\to T_2) = U(C)}$ ← funkcija poti

$\vec{\nabla}\times\vec{E} = 0 \;\wedge\; \frac{\partial\vec{B}}{\partial t} = 0$

d) Potenciali: $\boxed{\vec{B} = \vec{\nabla}\times\vec{A}} \Rightarrow \vec{\nabla}\vec{B} = 0$

$\vec{E} = -\vec{\nabla}\Phi \Rightarrow \vec{\nabla}\times\vec{E} = 0$ //

$\vec{\nabla}\times\vec{E} = -\frac{\partial}{\partial t}\vec{B} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{A}) = \vec{\nabla}\times\left(-\frac{\partial\vec{A}}{\partial t}\right)$ ✓

$$\Rightarrow \boxed{\vec{E} = -\vec{\nabla}\Phi - \frac{\partial\vec{A}}{\partial t}}$$

IV/6.:

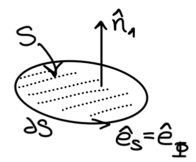
Za en ovoj: $\underline{-\oint_{\partial S}\vec{E}\,d\vec{s}} = -\int_{S_1}(\vec{\nabla}\times\vec{E})\,d\vec{S}$ (Stokes)

$= \int_{S_1}\frac{\partial\vec{B}}{\partial t}\,d\vec{S}$ (Maxwell)

$= \frac{\partial}{\partial t}\int_{S_1}\vec{B}\,d\vec{S}$ $(S_1 \neq S_1(t))$

$= \underline{\frac{\partial}{\partial t}\Phi_B}$

$n_1 = (\hat{e}_s\cdot\hat{e}_\Phi)\,\hat{e}_z$



Spomnimo se: $\underline{\Phi_B} = LI(\hat{e}_j\cdot e_\Phi)(\hat{e}_j\,\hat{n}_ž)(\hat{e}_z\hat{n}_1)$; $L = \frac{\mu\mu_0 N^2 S_1}{l}$

$= LI(\hat{e}_j\,\hat{e}_\Phi)(\hat{e}_j\,\hat{n}_ž)(\hat{e}_z\,\hat{e}_z)(\hat{e}_s\,\hat{e}_\Phi)$

$= \underline{LI(\hat{e}_j\,\hat{e}_\Phi)(\hat{e}_j\,\hat{n}_ž)(\hat{e}_s\,\hat{e}_\Phi)}$

a) $\hat{n}_ž = +\hat{e}_s$:

$(\hat{e}_j\hat{e}_\Phi)\cdot(\hat{e}_j\hat{e}_s)(\hat{e}_s\hat{e}_\Phi) = +1$

$\hat{e}_j, \hat{e}_\Phi, \hat{e}_s$ $\rightrightarrows$ $\quad$ $\leftrightarrows$ $\hat{e}_j, \hat{e}_s, \hat{e}_\Phi$

b) $\hat{n}_ž = -\hat{e}_s$:

$(\hat{e}_j\hat{e}_\Phi)(\hat{e}_j(-\hat{e}_s))(\hat{e}_s\hat{e}_\Phi) = -1$

$$\Rightarrow \Phi_B = \begin{cases} LI\,; & \hat{n}_ž = \hat{e}_s \\ -LI\,; & \hat{n}_ž = -\hat{e}_s \end{cases}$$

---

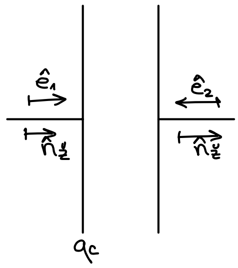
2. KIRCHHOFFOV IZREK (drugič)

- spremenljiva U in I
- C, L

a) Napetost na L:
- napetost odvisna od poti: skozi tuljavo
- žice tuljave: idealno prevodne: $\zeta \to 0 \Rightarrow \vec{E} = \zeta\vec{j} \to \vec{E} \to 0$

$$\Rightarrow \boxed{U_L = -\int_C \vec{E}\,d\vec{s} = 0}$$

b) Napetost na kondenzatorju:



- izberemo $\hat{n}_{\check{z}}$
- $q_c \equiv$ naboj na plošči, proti kateri kaže $\hat{n}_{\check{z}}$
- III/4: $I = \frac{dq_c}{dt}$
- izberemo smer obhoda

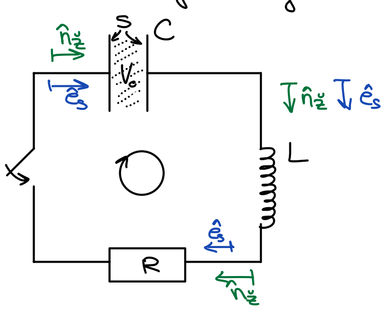
$$U_c = -\int \vec{E}\, d\vec{s}$$

od ene do druge plošče

- $\hat{e}_1$: $q_c > 0 \Rightarrow \vec{E}$ (→); $\vec{E}\hat{e}_1 > 0$
  $U_c < 0 \Rightarrow \boxed{U_c = -\frac{q_c}{C}}$
  $q_c < 0 \Rightarrow \vec{E}$ (←); $\vec{E}\hat{e}_1 < 0$
  $U_c > 0 \Rightarrow \boxed{U_c = -\frac{q_c}{C}}$
- $\hat{e}_2$: $\boxed{U_c = \frac{q_c}{C}}$

$$\Rightarrow \boxed{U_c = \begin{cases} -\frac{q_c}{C}\,; & \hat{e}_1\hat{n}_{\check{z}} = 1 \\ \frac{q_c}{C}\,; & \hat{e}_1\hat{n}_{\check{z}} = -1 \end{cases}}$$

Primer: električni nihajni krog



$t = 0$: $q_c(t=0) = q_0$
$I(t=0) = 0$

$$-\oint_{\partial S} \vec{E}\, d\vec{s} = \frac{d}{dt}\Phi_B$$

$$\boxed{U_c + U_L + U_R = \frac{d}{dt}\Phi_B} \quad (\Sigma U \neq 0)$$

$$U_L = 0;\ U_c = -\frac{q_c}{C};\ U_R = -IR;\ \frac{d}{dt}\Phi_B = L\dot{I}$$

$$\rightarrow \boxed{-\frac{q_c}{C} - IR = L\dot{I}}\,; \quad \dot{I} = \ddot{q}_c$$

$$\rightarrow -\frac{q_c}{C} - \dot{q}_c R = L\ddot{q}_c$$

$$L \neq 0 \rightarrow \boxed{\ddot{q}_c + \beta\dot{q}_c + \omega_0^2 q_c = 0;}\quad \beta = \frac{R}{L},\ \omega_0^2 = \frac{1}{LC}$$

enačba dušenega nihanja

$$\hookrightarrow q_c = Ae^{-\frac{\beta}{2}t}\sin(\omega t + \delta)\,; \quad \omega = \sqrt{\omega_0^2 - \frac{\beta^2}{4}}$$

(1) $L = 0$: $\boxed{\dot{q}_c + \frac{q_c}{\tau} = 0}$; $\tau = RC$ [s]

$$q_c(t) = q_0 \exp\left\{-\frac{t}{\tau}\right\}$$

$$\Rightarrow I = \dot{q}_c = -\frac{q_0}{\tau}\exp\left\{-\frac{t}{\tau}\right\}$$

Spomnimo se: $P_{el} = I_R^2 R = RI_0^2 \exp\left\{-\frac{2t}{\tau}\right\}$

$$A_{el} = \int_0^\infty P_{el}\, dt = \dots = \frac{q_0^2}{2C} \quad \Rightarrow \Delta W_n > 0 \rightarrow \Delta W_R > 0$$

$$\Delta W = 0 \Rightarrow \Delta W_c + \Delta W_R = 0 \rightarrow \Delta W_c = -\Delta W_R = -\frac{q_0^2}{2C}$$

$$W_c := \frac{q_c^2}{2C} = \frac{1}{2}\frac{(CU_c)^2}{C} = \frac{CU_c^2}{2} = \frac{1}{2}\frac{\varepsilon_0 S E^2 \ell^2}{\ell} = \frac{1}{2} S\ell\varepsilon_0 E^2 = \frac{1}{2} V\varepsilon_0 E^2 \quad (V = V_c \approx V_E)$$

volumen elektičnega polja

$$= \boxed{\frac{1}{2} V_E \varepsilon_0 E^2 =: W_E}$$

$$\boxed{W_E = V_E \frac{1}{2}\varepsilon\varepsilon_0 E^2}$$

(2) $R=0$: $L\ddot{q}_c + \frac{q_c}{C} = 0 \rightarrow \boxed{\ddot{q}_c + \omega_0^2 q_c = 0}$; $\omega_0^2 = \frac{1}{LC}$ $[s^{-2}]$

ENAČBA NEDUŠENEGA NIHANJA

$$q_c = A\sin(\omega_0 t + \delta)$$
$$I = A\omega_0\cos(\omega_0 t + \delta)$$

(1) $q_c(t=0) = q_0 \Rightarrow q_0 = A\sin\delta$

(2) $I(t=0) = 0 \Rightarrow 0 = A\omega_0\cos\delta$

$\Rightarrow \omega_0\cot\delta = 0 \Rightarrow \boxed{\delta = \frac{\pi}{2}}$

$\Rightarrow \boxed{q_0 = A}$

$$\Rightarrow \boxed{\begin{aligned} q_c &= q_0\cos(\omega_0 t) \\ I &= -I_0\sin(\omega_0 t) \end{aligned}}\ ; \quad I_0 = q_0\omega_0$$

$t=0$: $W_c(=W_E) = \frac{1}{2}\frac{q_0^2}{C}$

$t=\frac{\pi}{2\omega_0}$: $W_c(=W_E) = 0$

$\Rightarrow \Delta W_c = -\frac{1}{2}\frac{q_0^2}{C}$

$\Delta W = 0$

$\Delta W = \Delta W_c + \Delta W_L = 0$

$\Rightarrow \Delta W_L = -\Delta W_c = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}\frac{I_0^2}{C\omega_0^2} = \frac{1}{2}\frac{LCI_0^2}{C} = \frac{1}{2}LI_0^2$

$B = \frac{\mu_0\mu NI}{\ell} \rightarrow I = \frac{B\ell}{\mu\mu_0 N} \longrightarrow$

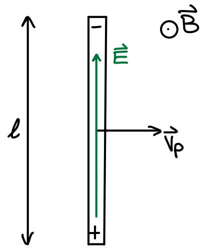
$\Rightarrow W_L = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu\mu_0 N^2 S_1}{\ell}\frac{\ell^2 B^2}{(\mu\mu_0)^2 N^2} = \frac{1}{2}S_1\ell\frac{B^2}{\mu\mu_0} = \boxed{\frac{1}{2}V_B\frac{B^2}{\mu\mu_0} = W_B}$

prostornina magnetnega polja

Gostota energij: $\boxed{w_E := \frac{W_E}{V_E} = \frac{1}{2}\varepsilon\varepsilon_0 E^2}$

$\boxed{w_B = \frac{W_B}{V_B} = \frac{1}{2}\frac{B^2}{\mu\mu_0}}$

"INDUKCIJA" (NAVIDEZNA INDUKCIJA) (Laurentsova sila)



$\vec{B} = B\hat{e}_z$; $B>0$

$\vec{v}_p = v_p\hat{e}_x$; $v_p > 0$

e^-: $q_1 = -q_0$

$$\boxed{\vec{F}_m = q\vec{v}\times\vec{B} = -q_0 v_p B\,\hat{e}_x\times\hat{e}_z = q_0 v_p B\hat{e}_y}$$

$$\boxed{\vec{F}_{el} = q\vec{E} = -q_0 E\hat{e}_y = q_0 E(-\hat{e}_y)}$$

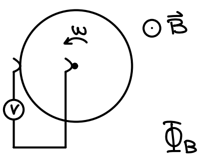
$\vec{F}_m + \vec{F}_{el} = 0$

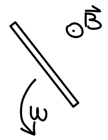
$q_0 v_p B\hat{e}_y + q_0 E(-\hat{e}_y) = 0 \rightarrow \boxed{E = v_p B} \rightarrow \boxed{U = \pm E\ell = \pm v_p B\ell}$

$\boxed{U = \vec{v}_p(\vec{B}\times\vec{\ell})\ ;\ \vec{\ell} = \ell\hat{e}_\ell}$

$\boxed{U = \int\vec{v}_p(\vec{B}\times d\vec{\ell})}$

Faradejev disk:

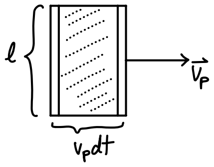




$\boxed{U = \frac{1}{2}\omega B\ell^2}$

$\Phi_B \neq \Phi_B(t)$

$\Rightarrow \frac{d\Phi_B}{dt} = 0$



$dS = l v_p dt$

$d\Phi_B = B\,dS = v_p B l\,dt \qquad v_p B l = \frac{d\Phi_B}{dt} = U \ (\neq U_i)$

$$-\oint_{\partial S} \vec{E}\,d\vec{s} = \int_S \frac{\partial \vec{B}}{\partial t}\,d\vec{S} \qquad S = S(t)$$

$$U_i = \int \frac{\partial \vec{B}}{\partial t}\,d\vec{S} \ ; \quad S \neq S(t)$$
$$= \frac{d}{dt}\int \vec{B}\,d\vec{S} = \frac{d}{dt}\Phi_B$$

ELEKTROMAGNETNO VALOVANJE

Maxwellove enačbe:
$$\vec{\nabla}\vec{B} = 0 \qquad \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla}\vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \frac{1}{c_0^2}\frac{\partial \vec{E}}{\partial t} \ ; \quad c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \approx 3\cdot 10^8\ \frac{m}{s}$$

Za prazen prostor:
$$\left.\begin{array}{l}\rho = 0 \\ \vec{j} = 0\end{array}\right\} \Rightarrow \begin{array}{ll}\vec{\nabla}\vec{B} = 0 & \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla}\vec{E} = 0 & \vec{\nabla}\times\vec{B} = \frac{1}{c_0^2}\frac{\partial \vec{E}}{\partial t}\end{array}$$

Matematična resnica: $\vec{E}$ poljuben: $\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\vec{\nabla}\vec{E}) - \nabla^2\vec{E}$; $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

$$\rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\vec{\nabla}\times\left(\frac{\partial \vec{B}}{\partial t}\right)$$

$$\underbrace{\vec{\nabla}(\vec{\nabla}\vec{E})}_{\overset{!!}{0}} - \nabla^2\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$
$$= -\frac{\partial}{\partial t}\left(\frac{1}{c_0^2}\frac{\partial \vec{E}}{\partial t}\right) \Rightarrow -\nabla^2\vec{E} = -\frac{1}{c_0^2}\frac{\partial^2\vec{E}}{\partial t^2} \rightarrow \boxed{\nabla^2\vec{E} = \frac{1}{c_0^2}\frac{\partial^2\vec{E}}{\partial t^2}}$$

$$\rightarrow \boxed{\nabla^2\vec{B} = \frac{1}{c_0^2}\frac{\partial^2\vec{B}}{\partial t^2}}$$

Komentarji:

1) elektromagnetno valovanje (potovanje motnje v $\vec{E}$ in $\vec{B}$) se lahko širi skozi prazen prostor

2) c_0 ... hitrost razširjanja motnje
$c_0 = $ konst. (!?)
↳ ne ujema se z Galilejevimi transformacijami: $S: \vec{v} = c_0\hat{e}_x$
$S': \vec{v}_0 = v_0\hat{e}_x$
GT: $\vec{v}' = \vec{v} - \vec{v}_0 = (c_0 - v_0)\hat{e}_x =$
$= c_0(1-\beta_0)\hat{e}_x$; $\beta_0 = \frac{v_0}{c_0}$

$\vec{E}\perp\vec{B}\perp\vec{c}_0$ (EM valovanje: transverzalno)

$B = \frac{E}{c_0}$

$w_E = \frac{1}{2}\varepsilon_0 E^2 \qquad w_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{E^2}{\mu_0 c_0^2} = \frac{\varepsilon_0\mu_0 E^2}{2\mu_0} = \frac{1}{2}\varepsilon_0 E^2 = w_E$

$w = w_E + w_B = \varepsilon_0 E^2$

$\vec{j}_{EM} = w\vec{c}_0 = \varepsilon_0 E^2\vec{c}_0$; $\vec{P} = \vec{E}\times\frac{\vec{B}}{\mu_0} = \vec{j}$
↑ Poyntingov vektor

6. POSEBNA TEORIJA RELATIVNOSTI

INVENTURA

Maxwellove enačbe:
$$\vec{\nabla}\vec{B}=0$$
$$\vec{\nabla}E=\frac{\rho}{\varepsilon_0}$$
$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{B}=\mu_0\vec{j}+\frac{1}{c_0^2}\frac{\partial\vec{E}}{\partial t}$$

Ohmov zakon: $\vec{E}=\zeta\vec{j}$

Lorentzova sila: $\vec{F}_L=qc_0\left(\frac{\vec{E}}{c_0}+\vec{\beta}_z\times\vec{B}\right)$; $\vec{\beta}_z=\frac{\vec{v}_q}{c_0}$

Galilejeve transformacije: $S\to S'$, $\vec{v}_0$
$$c_0t'=c_0t$$
$$\vec{r}'=\vec{r}-\vec{\beta}_0c_0t\ ;\ \vec{\beta}_0=\frac{\vec{v}_0}{c_0}$$

Transformacije $\vec{E}$ in $\vec{B}$:
$$\frac{\vec{E}'}{c_0}=\frac{\vec{E}}{c_0}+\vec{\beta}_0\times\vec{B}$$
$$\vec{B}'=\vec{B}-\vec{\beta}_0\times\frac{\vec{E}}{c_0}$$

TEŽAVE: • $\vec{F}_L'\neq\vec{F}_L$ (težave v linearnem približku glede na $\vec{\beta}_0$ ni)

• EM valovanje: hitrost $=c_0=\frac{1}{\sqrt{\mu_0\varepsilon_0}}=$ konst.
(v nasprotju z Galilejevimi transformacijami že v linearnem približku)

GALILEJEVE TRANSFORMACIJE (ponovitev)

S, S'; $\vec{v}_0=$ konst.
nepospešena sistema

$$\left.\begin{aligned}\vec{r}&: O\to T\\ \vec{r}'&: O'\to T\end{aligned}\right\}\ \vec{r}'=\vec{r}-\vec{v}_0t=\boxed{\begin{aligned}&\vec{r}-\vec{\beta}_0c_0t=\vec{r}'\\ &c_0t'=c_0t\end{aligned}}$$

$\boxed{\vec{v}_0=v_0\hat{e}_x}$ (gibanje sistema vzdolž x-osi glede na drugega)

$$\boxed{\begin{aligned}c_0t'&=c_0t\\ x'&=x-\beta_0c_0t\\ y'&=y\\ z'&=z\end{aligned}}\ ;\ \beta_0=\frac{v_0}{c_0}$$

4-dimenzionalni vektor prostor – čas: $^4X=\begin{bmatrix}c_0t\\ \vec{r}\end{bmatrix}=\begin{bmatrix}c_0t\\ x\\ y\\ z\end{bmatrix}$; $^4X'=\begin{bmatrix}c_0t'\\ \vec{r}'\end{bmatrix}=\begin{bmatrix}c_0t'\\ x'\\ y'\\ z'\end{bmatrix}$

$$G=\begin{bmatrix}1&0&0&0\\ -\beta_0&1&0&0\\ 0&0&1&0\\ 0&0&0&0\end{bmatrix}\qquad G^{-1}=\begin{bmatrix}1&0&0&0\\ \beta_0&1&0&0\\ 0&0&1&0\\ 0&0&0&0\end{bmatrix}$$

$$GG^{-1}=G^{-1}G=\mathbb{1}_{4\times4}$$

$$\boxed{^4X=G^{-1}\,{}^4X'}$$

GALILEJEVE TRANSFORMACIJE HITROSTI:

$$\vec{v}=\frac{d\vec{r}}{dt}=\begin{bmatrix}v_x\\ v_y\\ v_z\end{bmatrix};\ v_x=\frac{dx}{dt},\ v_y=\frac{dy}{dt},\ v_z=\frac{dz}{dt}$$

$$\Rightarrow v_x'=\frac{dx'}{dt'}=\frac{dx'}{dt}=\frac{d}{dt}(x-\beta_0c_0t)=v_x-\beta_0c_0=v_x-v_0$$
$$v_y'=\frac{dy'}{dt'}=\frac{dy'}{dt}=\frac{dy}{dt}=v_y\qquad v_z'=v_z$$

Primer: $S: \vec{v} = c_0 \hat{e}_x$, $\boxed{v = c_0} = v_x$

S': GT: $v_x' = c_0 - v_0$

$v_y' = v_y = 0$

$v_z' = v_z = 0$

$\boxed{v' = v_x' = c_0 - v_0} \boxed{= c_0(1-\beta_0)}$

2x narobe: ∘ Maxwellove enačbe $\not\ast$

∘ eksperimenti $\not\ast$

LORENTZOVE TRANSFORMACIJE

$$\boxed{\begin{aligned} c_0 t' &= \gamma_0 (c_0 t - \beta_0 x) \\ x' &= \gamma_0 (x - \beta_0 c_0 t) \\ y' &= y \\ z' &= z' \end{aligned}}$$

← LORENTZOVE TRANSFORMACIJE

$\beta_0 = \frac{v_0}{c_0}$ in $\gamma_0 = \frac{1}{\sqrt{1-\beta_0^2}}$

KOMENTARJI:

(1) $\beta_0^2 \geq 0 \Rightarrow \gamma_0 \geq 1$

(a) $v_0 \to c_0 \Rightarrow \beta_0 \to 1 \Rightarrow \gamma_0 \to \infty$

(b) $v_0 \ll c_0 \Rightarrow \beta_0^2 \ll 1 \Rightarrow \gamma_0 = \frac{1}{\sqrt{1-\beta_0^2}} \approx \frac{1}{1-\frac{1}{2}\beta_0^2} \approx 1 + \frac{1}{2}\beta_0^2 \Rightarrow \gamma_0 - 1 \approx \frac{1}{2}\beta_0^2$

$\boxed{c_0 t' =} \gamma_0 (c_0 t - \beta_0 x) = c_0 t - \beta_0 x + (\gamma_0 - 1)(c_0 t - \beta_0 x)$

$\approx c_0 t - \beta_0 x + \frac{1}{2}\beta_0^2 (c_0 t - \beta_0 x)$

$\boxed{\approx c_0 t - \beta_0 x}$ ← zaradi tega

$\boxed{x' \approx x - \beta_0 c_0 t}$ pogosta napaka

Lorentzove transformacije v linearnem približku

(2) $c_0 t = \gamma_0 (c_0 t - \beta_0 x) \to t' = \gamma_0 (t - \frac{\beta_0}{c_0} x) \neq t$

MATRIČNI ZAPIS:

$$\Lambda = \begin{bmatrix} \gamma_0 & -\beta_0\gamma_0 & 0 & 0 \\ -\beta_0\gamma_0 & \gamma_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \Lambda^{-1} = \begin{bmatrix} \gamma_0 & \beta_0\gamma_0 & 0 & 0 \\ \beta_0\gamma_0 & \gamma_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{{}^4X' = \Lambda\, {}^4X}$ in $\boxed{{}^4X = \Lambda^{-1}\, {}^4X'}$

Definicija: 4-dimenzionalni vektor je VEKTOR-ČETVEREC ali ŠTIRIVEKTOR, če se transformira pri prehodu $S \to S'$ z množenjem z Λ.

$\Lambda \to \Lambda^{-1} \Leftrightarrow \vec{v}_0 \to -\vec{v}_0$

$$\boxed{\begin{aligned} c_0 t &= \gamma_0 (c_0 t' + \beta_0 x') \\ x &= \gamma_0 (x' + \beta_0 c_0 t') \\ y &= y' \\ z &= z' \end{aligned}}$$

LORENTZOVE TRANSFORMACIJE HITROSTI:

$v_x = \frac{dx}{dt} \Rightarrow v_x' = \frac{dx'}{dt'} = \frac{dx'}{dt}\frac{dt}{dt'} = \frac{dx'}{dt} / \frac{dt'}{dt}$

$\left.\begin{aligned} \frac{dx'}{dt} &= \frac{\partial x'}{\partial t} + \frac{\partial x'}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial x'}{\partial t} + v_x \frac{\partial x'}{\partial x} \\ x' &= \gamma_0 (x - \beta_0 c_0 t) \Rightarrow \frac{\partial x'}{\partial t} = -\gamma_0 \beta_0 c_0 = -\gamma_0 v_0 ; \quad \frac{\partial x'}{\partial x} = \gamma_0 \end{aligned}\right\} \Rightarrow \boxed{\gamma_0 (v_x - v_0) = \frac{dx'}{dt}}$

$\left.\begin{aligned} \frac{dt'}{dt} &= \frac{\partial t'}{\partial t} + \frac{\partial t'}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial t'}{\partial t} + v_x \frac{\partial t'}{\partial x} \\ t' &= \gamma_0 (t - \frac{\beta_0}{c_0} x) \Rightarrow \frac{\partial t'}{\partial t} = \gamma_0 ; \quad \frac{\partial t'}{\partial x} = -\frac{\gamma_0 v_0}{c_0} \end{aligned}\right\} \Rightarrow \boxed{\frac{dt'}{dt} = \gamma_0 \left(1 - \frac{v_x v_0}{c_0^2}\right)}$

$\Rightarrow \boxed{v_x' = \frac{v_x - v_0}{1 - \frac{v_x v_0}{c_0^2}}}$

$$\boxed{v_y' = \frac{dy'}{dt'} = \frac{dy'}{dt}\frac{dt}{dt'} = \frac{dy'}{dt}\Big/\frac{dt'}{dt} = \frac{dy}{dt}\Big/\frac{dt'}{dt} = \frac{1}{\gamma_0}\frac{v_y}{\left(1-\frac{v_x v_0}{c_0^2}\right)}}$$

$$\boxed{v_z' = \frac{1}{\gamma_0}\frac{v_z}{\left(1-\frac{v_x v_0}{c_0^2}\right)}}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$\boxed{v'^2 = v_x'^2 + v_y'^2 + v_z'^2} = \left[(v_x - v_0)^2 + \frac{v_y^2+v_z^2}{\gamma_0^2}\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2 = \left[v_x^2 + v_0^2 - 2v_x v_0 + (1-\beta_0^2)(v_y^2+v_z^2)\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2 =$$

$$= \left[v_x^2 + v_y^2 + v_z^2 - \beta_0^2(v_y^2+v_z^2) + v_0^2 - 2v_x v_0\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2 =$$

$$= \left[v^2 - \beta_0^2 v_x^2 + \beta_0^2 v_x^2 - \beta_0^2(v_y^2+v_z^2) + v_0^2 - 2v_x v_0\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2 =$$

$$= \left[v^2(1-\beta_0^2) + v_x^2\beta_0^2 - 2v_x v_0 + v_0^2\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2 =$$

$$\beta^2 = \frac{v^2}{c_0^2} \longrightarrow = c_0^2\left[\beta^2(1-\beta_0^2) + \frac{v_x^2 v_0^2}{c_0^4} - \frac{2v_x v_0}{c_0^2} + \frac{v_0^2}{c_0^2}\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2 =$$

$$= c_0^2\left[\beta^2(1-\beta_0^2) + \frac{v_x^2 v_0^2}{c_0^4} - \frac{2v_x v_0}{c_0^2} + 1 - 1 + \beta_0^2\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2$$

$$= c_0^2\left[\beta^2(1-\beta_0^2) + \left(1-\frac{v_x v_0}{c_0^2}\right)^2 - (1-\beta_0^2)\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2$$

$$= c_0^2\left[(1-\beta_0^2)(\beta^2-1) + \left(1-\frac{v_x v_0}{c_0^2}\right)^2\right]\Big/\left(1-\frac{v_x v_0}{c_0^2}\right)^2$$

$$\boxed{= c_0^2\left[1 - \frac{(1-\beta^2)(1-\beta_0^2)}{\left(1-\frac{v_x v_0}{c_0^2}\right)^2}\right]}$$

Primer: S: $v = c_0$; $v' = ?$

$$\Rightarrow \beta^2 = \frac{v^2}{c_0^2} = 1 \Rightarrow 1-\beta^2 = 0$$

$$\Rightarrow v'^2 = c_0^2\left[1 - \frac{(1-\beta^2)(1-\beta_0^2)}{\left(1-\frac{v_x v_0}{c_0^2}\right)^2}\right] = c_0^2 \rightarrow v' = c_0$$

∄ ABSOLUTNI ČAS

$$c_0 t' = \gamma_0(c_0 t - \beta_0 x) \Rightarrow t' = \gamma_0\left(t - \frac{\beta_0 x}{c_0}\right) \Rightarrow \text{v splošnem } t' \neq t$$

Primer: $\mu^\pm$: (mion)

S: mirovni sistem $\mu^\pm$

$t_1 = 0$	$t_2 = \tau \approx 2\cdot 10^{-6}$ s	$c_0\tau \approx 600$ m
$x_1 = 0$	$x_2 = 0$	
$y_1 = 0$	$y_2 = 0$	
$z_1 = 0$	$z_2 = 0$	$\rightarrow \underset{2\mu s}{\Delta t} = t_2 - t_1 = \tau$

$$\gamma_0 = \frac{1}{\sqrt{1-\beta_0^2}} > 1$$

S': $\vec{v}_0 = v_0\hat{e}_x$ glede na S

$$\underbrace{t_1' = \gamma_0\left(t_1 - \frac{\beta_0}{c_0}x\right) = 0 \qquad t_2' = \gamma_0\left(t_2 - \frac{\beta_0}{c_0}x_2'\right) = \gamma_0 t_2}_{} \qquad \rightarrow \underset{2\mu s}{\Delta t'} = t_2' - t_1' = t_2' = \gamma_0\tau$$

$LT \Rightarrow \Delta t' > \Delta t$ (relativistično podaljšanje časa)

$\Delta x \lessapprox c_0 \Delta t$ $(\Delta t = \Delta t')$ GT

c_0 in KAVZALNOST:

S: t_1 t_2 $\quad t_2 > t_1 \Rightarrow \boxed{\Delta t = t_2 - t_1 > 0}$

x_1 x_2

$y_1 = 0$ $y_2 = 0$

$z_1 = 0$ $z_2 = 0$

$$S': \left.\begin{array}{l} t_1' = \gamma_0 (t_1 - \frac{\beta_0}{c_0} x_1) \\ t_2' = \gamma_0 (t_2 - \frac{\beta_0}{c_0} x_2) \end{array}\right\} \Rightarrow \Delta t' = t_2' - t_1' = \gamma_0 \Big[\underbrace{t_2 - t_1}_{\Delta t} - \frac{\beta_0 \Delta x}{c_0}\Big] \; ; \; \Delta x = x_2 - x_1$$

$$\boxed{\Delta t' < 0} \; ; \text{ če } \; \Delta t - \beta_0 \frac{\Delta x}{c_0} < 0 \rightarrow \boxed{\Delta t < \frac{\beta_0 \Delta x}{c_0}}$$

t_1: A ustreli ; v

$t_2 = t_1 + \Delta t$: zadene B $\quad \boxed{\Delta t = \frac{\Delta x}{v}}$

$$\Delta t' < 0 \Rightarrow \Delta t < \frac{\beta_0 \Delta x}{c_0} \rightarrow \frac{\Delta x}{v} < \frac{\beta_0 \Delta x}{c_0} \Rightarrow \boxed{v > \frac{1}{\beta_0} c_0 > c_0} \quad \beta_0 < 1$$

kršitev kavzalnosti

C: $t_2' (< t_1')$ vidi: B pade

∄ ABSOLUTNA RAZDALJA

S: $x_1 = 0$ ob (vsakem) času t_1

$x_2 = \ell$ ob (vsakem) času t_2 ; ℓ... dolžina rakete v sistemu S

$$D = |\vec{r}_1(t_1) - \vec{r}_2(t_2 = t_1)| = |x_1(t_1) - x_2(t_2 = t_1)| = \ell$$

S': $D' = |\vec{r}_1'(t_1') - \vec{r}_2'(t_2' = t_1')|$

$$c_0 t_1' = \gamma_0 (c_0 t_1 - \beta_0 \cancel{x_1}) = \gamma_0 c_0 t_1 \qquad c_0 t_2' = \gamma_0 (c_0 t_2 - \beta_0 \ell)$$

$$x_1' = \gamma_0 (\cancel{x_1} - \beta_0 c_0 t_1) = -\gamma_0 v_0 t_1 \qquad x_2' = \gamma_0 (\ell - v_0 t_2)$$

$$y_1' = y_1 \qquad y_2' = y_2$$

$$z_1' = z_1 \qquad z_2' = z_2$$

$\rightarrow \boxed{D' = |x_1'(t_1') - x_2'(t_2' = t_1')|}$ $\quad t_2' = t_1'$ ... razdalja med koncema rakete ob istem času

$$c_0 t_2' = c_0 t_1'$$

$$\gamma_0 (c_0 t_2 - \beta_0 \ell) = \gamma_0 (c_0 t_1)$$

$$c_0 t_1 = c_0 t_2 - \beta_0 \ell \longrightarrow \boxed{t_2 = t_1 + \frac{\beta_0}{c_0} \ell}$$

$$\boxed{x_1'(t_1) = -\gamma_0 c_0 t_1} \quad \boxed{x_2'(t_2' = t_1') = \gamma_0 (\ell - v_0 t_2) = \gamma_0 \Big(\ell - v_0 \big(t_1 + \frac{\beta_0 \ell}{c_0}\big)\Big) = \gamma_0 \ell (1 - \beta_0^2) - \gamma_0 v_0 t_1 =}$$

$$= \frac{\gamma_0 \ell}{\gamma_0^2} + x_1'(t_1') = \boxed{\frac{\ell}{\gamma_0} + x_1'(t_1')}$$

$$\rightarrow \boxed{D' = \Big| x_1'(t_1') - \frac{\ell}{\gamma_0} - x_1'(t_1') \Big| = \frac{\ell}{\gamma_0}} \; ; \; \gamma_0 > 1 \Rightarrow \boxed{D' < D}$$

SKALARJI V PTR

Definicija: skalar ≡ invarianta na LT

$\vec{r}$ ... krajevni vektor T

$r = \sqrt{\vec{r}\vec{r}} = \sqrt{\langle \vec{r}, \vec{r} \rangle}$

O: $O^T = O^{-1}$

$$\vec{r}' = O\vec{r} \Rightarrow r'^2 = \langle \vec{r}', \vec{r}' \rangle = \langle O\vec{r}, O\vec{r} \rangle = \langle \vec{r}, O^T O \vec{r} \rangle = \langle \vec{r}, \vec{r} \rangle = r^2$$

$$r^2 = \langle \vec{r}, \vec{r} \rangle = \langle \mathbb{1}_{3\times3} \vec{r}, \vec{r} \rangle \; ; \; \mathbb{1}_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; ; \text{ evklidski prostor}$$

$$\Lambda = \begin{bmatrix} \gamma_0 & -\beta_0\gamma_0 & 0 & 0 \\ -\beta_0\gamma_0 & \gamma_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \Lambda^T \neq \Lambda^{-1} = \begin{bmatrix} \gamma_0 & \beta_0\gamma_0 & 0 & 0 \\ \beta_0\gamma_0 & \gamma_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^4X' = \Lambda\, {}^4X$$

$${}^4X'^2 = \langle \mathbb{1}_{4\times4} {}^4X', {}^4X' \rangle = \langle \Lambda {}^4X, \Lambda {}^4X \rangle = \langle {}^4X, \underbrace{\Lambda^2}_{\Lambda^T\Lambda \neq \mathbb{1}_{4\times4}} {}^4X \rangle \neq \langle \mathbb{1}_{4\times4} {}^4X, {}^4X \rangle \neq {}^4X^2$$

$$\mathbb{1}_{4\times4} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \eta$$

$$\Lambda \eta \Lambda = \eta \Rightarrow \eta \Lambda = \Lambda^{-1} \eta$$

$$\langle \eta\, {}^4X', {}^4X' \rangle = \langle \eta \Lambda {}^4X, {}^4X \rangle = \langle \Lambda^T \eta \Lambda {}^4X, {}^4X \rangle = \langle \eta\, {}^4X, {}^4X \rangle$$

$$\eta\, {}^4X = \eta \cdot [c_0t \;\; x \;\; y \;\; z]^T = \begin{bmatrix} c_0t \\ -\vec{r} \end{bmatrix} = {}_4X$$

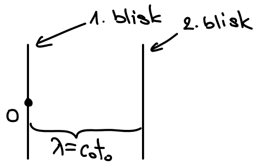
$$\rightarrow \boxed{\langle {}_4X, {}^4X \rangle = {}_4X \cdot {}^4X} \quad \rightarrow \boxed{{}_4X \cdot {}^4X = {}_4X' \cdot {}^4X'}$$

DOPLERJEV POJAV

- mehansko valovanje: v glede na snov ← ne deluje
- elektro-magnetno valovanje v praznem prostoru: relativna hitrost

S: mirovni sistem oddajnika

1. blisk: $t_1 = t$, $x_1 = 0$
2. blisk: $t_2 = t_1 + t_0$; $x_2 = 0$



S': mirovni sistem sprejemnika, ki se giblje s hitrostjo $\vec{v}_0 = v_0 \hat{e}_x$ glede na S

$$t_1' = \gamma_0 (t_1 - \tfrac{\beta_0}{c_0} x_1) = \gamma_0 t_1$$

$$x_1'(t_1) = \gamma_0 (x_1 - \beta_1 c_0 t_1) = -\gamma_0 v_0 t_1 = -v_0 t_1' = x_1'(t_1')$$

$$t_2' = \gamma_0 (t_2 - \tfrac{\beta_0}{c_0} x_2) = \gamma_0 t_2 = \gamma_0 (t_1 + t_0) = \gamma_0 t_1 + \gamma_0 t_0 = t_1' + \gamma_0 t_0$$

$$x_2'(t_2) = \gamma_0 (x_2 - \beta_0 c_0 t_2) = -\gamma_0 v_0 t_2 = -v_0 t_2' = x_2'(t_2')$$

tudi v S' se bliska gibljeta s c_0

$$\boxed{x_1'(t_2') = x_1'(t_1') + c_0 (t_2' - t_1') = -v_0 t_1' + c_0 \gamma_0 t_0}$$

$$\lambda' = |x_1'(t_2') - x_2'(t_2')| = |-v_0 t_1' + c_0 \gamma_0 t_0 + v_0 t_2'| = |-v_0 t_1' + c_0 \gamma_0 t_0 + v_0 (t_1' + \gamma_0 t_0)| = |\gamma_0 t_0 (c_0 + v_0)| = \frac{(c_0 + v_0) t_0}{\sqrt{1 - \frac{v_0^2}{c_0^2}}} =$$

$$= \frac{(c_0 + v_0)\, c_0 t_0}{\sqrt{c_0^2 - v_0^2}} = \boxed{\lambda \sqrt{\frac{c_0 + v_0}{c_0 - v_0}}}$$

$$\boxed{\nu' = \frac{c_0}{\lambda'} = \sqrt{\frac{c_0 \mp v_0}{c_0 \pm v_0}}\, \frac{c_0}{\lambda} = \sqrt{\frac{c_0 \mp v_0}{c_0 \pm v_0}}\, \nu}$$

ČETVEREC HITROSTI

Spomnimo se: $v'^2 = c_0^2 \left[1 - \frac{(1-\beta_0^2)(1-\beta^2)}{(1 - \frac{v_0 v_x}{c_0^2})^2} \right]$; $\beta^2 = \frac{v^2}{c_0^2}$; $\beta_0^2 = \frac{v_0^2}{c_0^2}$

$$\Rightarrow \frac{v'^2}{c_0^2} = 1 - \frac{(1-\beta_0^2)(1-\beta^2)}{(1 - \frac{v_0 v_x}{c_0^2})^2} \rightarrow 1 - \beta'^2 = \frac{(1-\beta_0^2)(1-\beta^2)}{(1 - \frac{v_x v_0}{c_0^2})^2}$$

$$\rightarrow \frac{1}{\gamma'^2} = \frac{1}{\gamma_0^2 \gamma^2 (1 - \frac{v_0 v_x}{c_0^2})^2}; \quad \gamma_0^2 = \frac{1}{1-\beta_0^2}; \; \gamma'^2 = \frac{1}{1-\beta'^2}; \; \gamma^2 = \frac{1}{1-\beta^2}$$

$$\Rightarrow \boxed{\frac{\gamma}{\gamma'} = \frac{1}{\gamma_0} \frac{1}{1 - \frac{v_0 v_x}{c_0^2}} = \frac{dt}{dt'}} \quad \left(= 1 / \frac{dt'}{dt}\right)$$

4D-hitrost: $\boxed{{}^4u := \gamma \frac{d}{dt}({}^4X)}$

$$\boxed{{}^4u' = \Lambda {}^4u} = \begin{bmatrix} \gamma' c_0 \\ \gamma' \vec{v}' \end{bmatrix}$$

Dokaz: ${}^4X' = \Lambda {}^4X$; $\Lambda \neq \Lambda(t, x)$ $\qquad \frac{\partial \Lambda}{\partial t} + \frac{\partial \Lambda}{\partial x} \frac{dx}{dt} = 0$

$$\frac{d}{dt}({}^4X') = \frac{d}{dt}(\Lambda {}^4X) = \Lambda \frac{d}{dt}({}^4X) + \left(\frac{d}{dt}\Lambda\right) \cdot {}^4X = \Lambda \frac{d}{dt}({}^4X)$$

$$\frac{d}{dt'}({}^4X') = \frac{d}{dt}({}^4X') \frac{dt}{dt'} = \Lambda \frac{d}{dt}({}^4X) \frac{\gamma}{\gamma'} = \frac{1}{\gamma'} \Lambda \gamma \frac{d}{dt}({}^4X)$$

$$\rightarrow \underbrace{\gamma' \frac{d}{dt'}({}^4X')}_{{}^4u'} = \underbrace{\Lambda \gamma \frac{d}{dt}({}^4X)}_{\Lambda {}^4u} \qquad \square$$

ČETVEREC POSPEŠKA

$$\left.\begin{matrix} {}^4u \\ \frac{d}{dt} \\ \times\gamma \end{matrix}\right\} \Rightarrow \boxed{{}^4b = \gamma\frac{d}{dt}({}^4u)}$$

$$\frac{d}{dt}\gamma = \frac{d}{dt}\frac{1}{\sqrt{1-\frac{v^2}{c_0^2}}} = \frac{d}{dt}\left(1-\frac{\vec v\vec v}{c_0^2}\right)^{-\frac12} = \frac12\left(1-\frac{\vec v\vec v}{c_0^2}\right)^{-\frac32}\cdot\frac{2\vec a\vec v}{c_0^2} = \gamma^3\frac{\vec a\vec v}{c_0^2}$$

$$\boxed{{}^4b = \begin{bmatrix}\gamma^4\frac{\vec a\vec v}{c_0^2}\\ \gamma^4\frac{\vec a\vec v}{c_0^2}\vec v+\gamma^2\vec a\end{bmatrix}}$$

ČETVERCI ${}^4P$, ${}^4F$, ${}^4j_e$

$$\boxed{{}^4P = m_0\,{}^4u} = \begin{bmatrix} m_0\gamma c_0 \\ m_0\gamma\vec v\end{bmatrix} \quad \leftarrow \text{ČETVEREC GIBALNE KOLIČINE}$$

masa v mirovnem sistemu

$$\boxed{{}^4F = m_0\,{}^4b} = \begin{bmatrix} m_0\gamma^4\frac{\vec a\vec v}{c_0^2} \\ m_0\gamma^4\frac{\vec a\vec v}{c_0^2}\vec v+\gamma^2\vec a\end{bmatrix} \quad \leftarrow \text{ČETVEREC SILE}$$

$$\boxed{{}^4j_e = \rho_0\,{}^4u} = \begin{bmatrix} \rho_0\gamma c_0 \\ \rho_0\gamma\vec v\end{bmatrix} \quad \leftarrow \text{ČETVEREC GOSTOTE ELEKTRIČNEGA TOKA}$$

${}^4P$: $m_0\gamma c_0 = \frac{1}{c_0}(mc_0^2\gamma) = \frac{E}{c_0} \quad \Rightarrow \quad \boxed{E = mc_0^2\gamma}$

$\qquad = \frac{1}{c_0}\left(mc_0^2\frac{1}{\sqrt{1-\beta^2}}\right)$

$$\beta\ll 1:\ \frac{1}{\sqrt{1-\beta^2}} \approx \frac{1}{1-\frac12\beta^2} \approx 1+\frac12\beta^2$$

$$m_0\gamma c_0 \approx \frac{1}{c_0}\left(m_0c_0^2\left(1+\frac12\beta^2\right)\right) = \frac{1}{c_0}\Big(\underbrace{m_0c_0^2}_{E_0} + \underbrace{\tfrac12 m_0v^2}_{W_k\,(=T)}\Big)$$

$$\boxed{{}^4P = \begin{bmatrix}\frac{E}{c_0}\\ \vec p\end{bmatrix}}$$

$$\vec p = m_0\gamma\vec v \qquad (v\ll c_0;\ \gamma\approx 1 \Rightarrow \vec p\approx m\vec v)$$

$$\vec p\vec p = m_0^2\gamma^2v^2$$

$$\Rightarrow\ p^2c_0^2 = m_0^2\gamma^2c_0^2v^2 = m_0^2c_0^4v^2\beta^2$$

$$E^2-p^2c_0^2 = m_0^2c_0^4\gamma^2(1-\beta^2) = m_0^2c_0^4 \quad \rightarrow \quad \boxed{E^2 = p_0^2c_0^2 + m_0^2c_0^4}$$

$${}_4P\cdot{}^4P = \left[\frac{E}{c_0}\ \ -\vec p\right]^T\cdot\left[\frac{E}{c_0}\ \ \vec p\right]^T = \frac{E^2}{c_0^2}-p^2 = \frac{1}{c_0^2}(E^2-p^2c_0^2) = \frac{1}{c_0^2}m_0^2c_0^4 = m_0^2c_0^2$$

$$E = m\gamma c_0^2 = mc_0^2\frac{1}{\sqrt{1-\beta^2}} \qquad v\to c_0 \Rightarrow \beta\to 1 \Rightarrow \gamma\to\infty \Rightarrow E\to\infty$$

Primer: (LEP)

elektron-volt

$e^\pm$; $m_0c_0^2 \approx 0{,}5\,\mathrm{MeV} = 0'5\cdot 10^6\cdot 1'6\cdot 10^{-19}\,\mathrm{J}$

$E = 100\,\mathrm{GeV} = 100\cdot 10^9\cdot 1'6\cdot 10^{-19}\,\mathrm{J}$

$$\Rightarrow\ \frac{mc_0^2}{E} = \frac{1}{\gamma} = 0'5\cdot 10^{-5}$$

$$\frac{1}{\gamma^2} = 0'25\cdot 10^{10} = 1-\beta^2$$

$$\beta^2 = 1-0'25\cdot 10^{10} \quad \Rightarrow \beta = 1-0'125\cdot 10^{10} \quad \Rightarrow v = c_0\beta$$

ČETVEREC LORENTZOVE SILE

$\boxed{{}^4F' = \Lambda\,{}^4F}$ $({}^4F' \neq {}^4F)$ $(\vec{F}' = \vec{F})$

$$\vec{F}_L = q(\vec{E} + \vec{v}_q \times \vec{B})$$

$$M = \begin{bmatrix} 0 & -\frac{E_x}{c_0} & -\frac{E_y}{c_0} & -\frac{E_z}{c_0} \\ \frac{E_x}{c_0} & 0 & -B_z & B_y \\ \frac{E_y}{c_0} & B_z & 0 & -B_x \\ \frac{E_z}{c_0} & -B_y & B_x & 0 \end{bmatrix}$$ (6 neodvisnih elementov)

$$_4u = \eta\,{}^4u = \begin{bmatrix} \gamma c_0 \\ -\gamma \vec{v} \end{bmatrix}$$

$\boxed{{}^4F_L = qM\,{}_4u}$ ← ČETVEREC LORENTZOVE SILE

$$= q \cdot \begin{bmatrix} \gamma \frac{\vec{E}\vec{v}}{c_0} \\ \gamma(\vec{E} + \vec{v} \times \vec{B}) \end{bmatrix} = \begin{bmatrix} F_{L,0} \\ \vec{F}_L \end{bmatrix}$$

$$\vec{F}_L = \gamma q(\vec{E} + \vec{v}_q \times \vec{B}); \quad |\vec{v}| \ll c_0 \Rightarrow \beta \ll 1 \Rightarrow \gamma \approx 1 + \frac{\beta^2}{2} = 1 + O(\beta^2)$$

$$\vec{v} \times \vec{B} = -\vec{B} \times \vec{v} = \vec{B} \times (-\vec{v}) = \begin{bmatrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{bmatrix}$$

$S \to S'$: ${}^4F_L' = \Lambda\,{}^4F_L$ $\qquad \Lambda\eta\Lambda = \eta \Rightarrow \eta\Lambda = \Lambda^{-1}\eta$

$q' = q$

$_4u' = \eta'\,{}^4u' = \eta\,{}^4u' = \eta\Lambda\,{}^4u = \Lambda^{-1}\eta\,{}^4u = \Lambda^{-1}\,{}_4u$

$\boxed{{}^4F_L' = q' M'\,{}_4u'}$ relativistični princip

$\Lambda\,{}^4F_L = qM'\Lambda^{-1}\,{}_4u$

$\xrightarrow{\Lambda^{-1}\cdot}$ $\boxed{{}^4F_L = q\,\Lambda^{-1}M'\Lambda^{-1}\,{}_4u}$

$\Rightarrow \Lambda^{-1}M'\Lambda^{-1} = M$

$\Rightarrow \boxed{M' = \Lambda M \Lambda}$ $(M$ tenzor$)$

$$M' = \begin{bmatrix} 0 & -\frac{E_x'}{c_0} & -\frac{E_y'}{c_0} & -\frac{E_z'}{c_0} \\ \frac{E_x'}{c_0} & 0 & -B_z' & B_y' \\ \frac{E_y'}{c_0} & B_z' & 0 & -B_x' \\ \frac{E_z'}{c_0} & -B_y' & B_x' & 0 \end{bmatrix}$$

$$\frac{E_x'}{c_0} = \frac{E_x}{c_0} \qquad B_x' = B_x$$
$$\frac{E_y'}{c_0} = \gamma_0\left(\frac{E_y}{c_0} - \beta_0 B_z\right) \qquad B_y' = \gamma_0\left(B_y + \beta_0 \frac{E_z}{c_0}\right)$$
$$\frac{E_z'}{c_0} = \gamma_0\left(\frac{E_z}{c_0} + \beta_0 B_y\right) \qquad B_z' = \gamma_0\left(B_z - \beta_0 \frac{E_y}{c_0}\right)$$

V vektorski obliki: $\frac{\vec{E}'}{c_0} = \gamma_0\left(\frac{\vec{E}}{c_0} + \vec{\beta}_0 \times \vec{B}\right) - (\gamma_0 - 1)\hat{e}_x\left(\frac{\vec{E}}{c_0}\cdot\hat{e}_0\right)$; $\hat{e}_0 = \hat{e}_x \wedge \vec{\beta}_0 = \beta_0\hat{e}_0 = \frac{v_0}{c_0}\hat{e}_0$

$$= \frac{\vec{E}}{c_0} + \vec{\beta}_0 \times \vec{B} + (\gamma_0 - 1)\left[\frac{\vec{E}}{c_0} + \vec{\beta}_0 \times \vec{B} - \hat{e}_0\left(\frac{\vec{E}}{c_0}\cdot\hat{e}_0\right)\right]$$

$\beta_0 \ll 1,\ \gamma_0 - 1 = O(\beta^2)$: $\approx \underline{\frac{\vec{E}}{c_0} + \vec{\beta}_0 \times \vec{B}}$

$$\vec{B}' = \gamma_0\left(\vec{B} - \vec{\beta}_0 \times \frac{\vec{E}}{c_0}\right) - (\gamma_0 - 1)\hat{e}_0(\vec{B}\hat{e}_0)$$

$$= \vec{B} - \vec{\beta}_0 \times \frac{\vec{E}}{c_0} + (\gamma_0 - 1)\left[\vec{B} - \vec{\beta}_0 \times \frac{\vec{E}}{c_0} - \hat{e}_0(\vec{B}\hat{e}_0)\right]$$

$\beta_0 \ll 1$: $\approx \underline{\vec{B} - \vec{\beta}_0 \times \frac{\vec{E}}{c_0}}$

MAXWELLOVE ENAČBE in PTR

$$N = \begin{bmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & \frac{E_z}{c_0} & -\frac{E_y}{c_0} \\ B_y & -\frac{E_z}{c_0} & 0 & \frac{E_x}{c_0} \\ B_z & \frac{E_y}{c_0} & -\frac{E_x}{c_0} & 0 \end{bmatrix}$$

$$N' = \begin{bmatrix} 0 & -B'_x & -B'_y & -B'_z \\ B'_x & 0 & \frac{E'_z}{c_0} & -\frac{E'_y}{c_0} \\ B'_y & -\frac{E'_z}{c_0} & 0 & \frac{E'_x}{c_0} \\ B'_z & \frac{E'_y}{c_0} & -\frac{E'_x}{c_0} & 0 \end{bmatrix} \qquad \boxed{N' = \Lambda N \Lambda} \qquad \text{(N tenzor)}$$

$$\partial_0 \equiv \frac{1}{c_0}\frac{\partial}{\partial t} \qquad {}_4\partial \equiv \begin{bmatrix} \partial_0 \\ \partial_1 \\ \partial_2 \\ \partial_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{c_0}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix}$$
$$\partial_1 \equiv \frac{\partial}{\partial x}$$
$$\partial_2 \equiv \frac{\partial}{\partial y}$$
$$\partial_3 \equiv \frac{\partial}{\partial z}$$

$$\boxed{{}_4\partial' = \Lambda^{-1}\,{}_4\partial} : \quad \partial'_0 = \frac{1}{c_0}\frac{\partial}{\partial t'} = \frac{1}{c_0}\left[\frac{\partial t}{\partial t'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial t'}\frac{\partial}{\partial x}\right] \; ; \quad t = \gamma_0\left(t' + \frac{v_0}{c_0^2}x'\right); \quad x = \gamma_0(x + v_0 t')$$
$$= \frac{v_0}{c_0}\left[\frac{\partial}{\partial t} + v_0\frac{\partial}{\partial x}\right]$$
$$= \gamma_0\left[\partial_0 + \beta_0\partial_1\right] \qquad \partial'_1 = \gamma_0\left[\partial_1 + \beta_0\partial_0\right]$$
$$\vdots$$

$$\boxed{{}^4j_e = \rho_0\,{}^4u = \begin{bmatrix} \rho_0\gamma c_0 \\ \rho_0\gamma\vec{v} \end{bmatrix} = \begin{bmatrix} \rho c_0 \\ \rho\vec{v} \end{bmatrix} = \begin{bmatrix} \rho c_0 \\ \vec{j}_e \end{bmatrix}} \; ; \quad \vec{j}_e = \rho\vec{v}$$

$${}_4\partial^T = \left[\partial_0 \;\; \partial_1 \;\; \partial_2 \;\; \partial_3\right]$$

$$S: \boxed{\begin{matrix} ({}_4\partial^T M)^T = \mu_0\,{}^4j \\ ({}_4\partial^T N)^T = {}^4 0 \end{matrix}} \leftarrow \text{MAXWELLOVE ENAČBE}$$

$$({}_4\partial^T N)^T = \left(\left[\frac{1}{c_0}\frac{\partial}{\partial t} \;\; \frac{\partial}{\partial x} \;\; \frac{\partial}{\partial y} \;\; \frac{\partial}{\partial z}\right]\cdot N\right)^T = \begin{bmatrix} \vec{\nabla}\vec{B} \\ -\frac{1}{c_0}\frac{\partial B_x}{\partial t} - \frac{1}{c_0}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) \\ -\frac{1}{c_0}\frac{\partial B_y}{\partial t} + \frac{1}{c_0}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) \\ -\frac{1}{c_0}\frac{\partial B_z}{\partial t} - \frac{1}{c_0}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right) \end{bmatrix} = \begin{bmatrix} \vec{\nabla}\vec{B} \\ -\frac{1}{c_0}\left(\frac{\partial\vec{B}}{\partial t} + \vec{\nabla}\times\vec{E}\right) \end{bmatrix} = 0$$

$$\rightarrow \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

(Podobno dobimo ostale Maxwellove enačbe.)

$$S': \boxed{({}_4\partial'^T M')^T = ({}_4\partial^T\Lambda^{-1}\Lambda M\Lambda)^T = ({}_4\partial^T M\Lambda)^T = \Lambda^T({}_4\partial^T M)^T = \Lambda\mu_0\,{}^4j = \mu_0\Lambda\,{}^4j = \mu_0\,{}^4j'}$$

$${}_4\partial'^T = (\Lambda^{-1}\,{}_4\partial)^T = {}_4\partial^T\Lambda^{-T} = {}_4\partial^T\Lambda^{-1}$$

ZAKLJUČEK

$${}^4X \equiv \begin{bmatrix} c_0 t \\ \vec{r} \end{bmatrix} \qquad S \to S': \boxed{{}^4X' = \Lambda\,{}^4X}$$

$${}^4u \equiv \gamma\frac{d}{dt}({}^4X) \qquad {}^4b \equiv \gamma\frac{d}{dt}({}^4u) \qquad {}^4P \equiv m_0\,{}^4u \qquad {}^4F \equiv m_0\,{}^4b \qquad {}^4j \equiv \rho_0\,{}^4u$$

$$\boxed{{}^4F_L = qM\,{}_4u} \qquad \boxed{({}_4\partial^T N)^T = 0}$$

$$\boxed{({}_4\partial^T M)^T = \mu_0\,{}^4j} \qquad \boxed{\vec{E} = \zeta\vec{j}}$$